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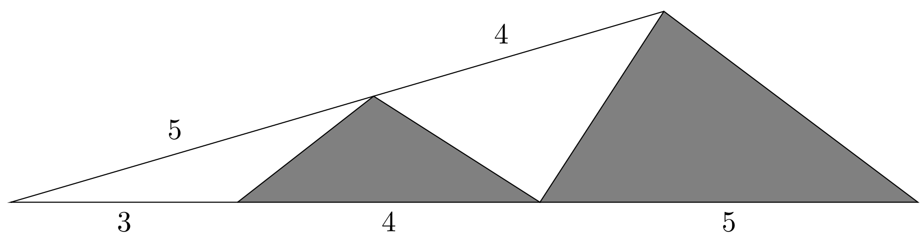
	The Area Method	
Western PA ARML Practice		*October 18, 2015*

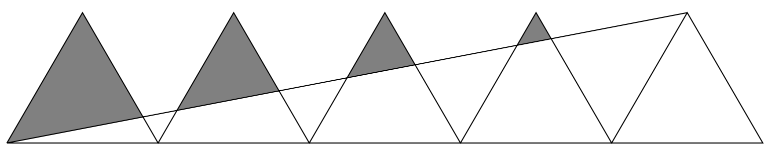
1 Warm-up problems

1. In square $ABCD$, line segments are drawn from A to the midpoint of BC, from B to the midpoint of CD, from C to the midpoint of DA, and from D to the midpoint of AB. The four segments form a smaller square within square $ABCD$. If $AB = 1$, what is the area of the smaller square?

2. In the diagram below, what is the ratio of the areas of the two shaded triangles?



3. In the diagram below, what is the ratio of the shaded area to the area of one of the five congruent triangles?



2 The area method

The fundamental tools of the area method are the following two lemmas:

Intersection Lemma. If segments AB and CD intersect at X, then $\frac{AX}{BX} = \frac{S_{ACD}}{S_{BCD}}$.

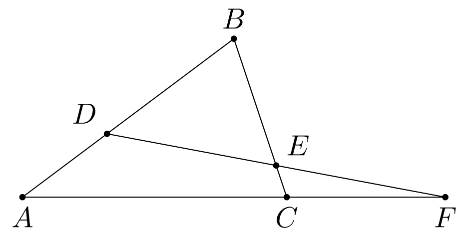
Vertex Sliding Lemma. If point C is on segment PQ, then $S_{ABC} = \frac{PC}{PQ} \cdot S_{ABQ} + \frac{CQ}{PQ} \cdot S_{ABP}$.

These are at their strongest when we allow areas and ratios to have signs, by the following rules:

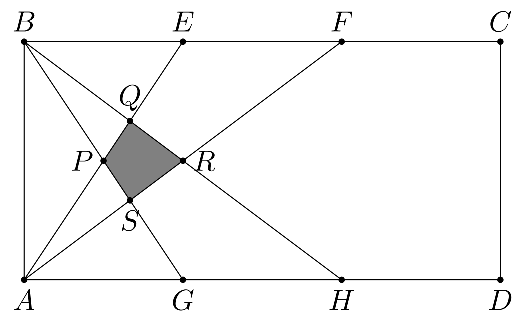
- $\frac{AB}{CD}$ is positive if AB and CD are pointing in the same direction, and negative otherwise.
- S_{ABC} is positive if A, B, C are in clockwise order around $\triangle ABC$, and negative otherwise.

If this seems overwhelming, then you can ignore signs (as long as AB does not intersect PQ in the second lemma), getting out slightly less information.

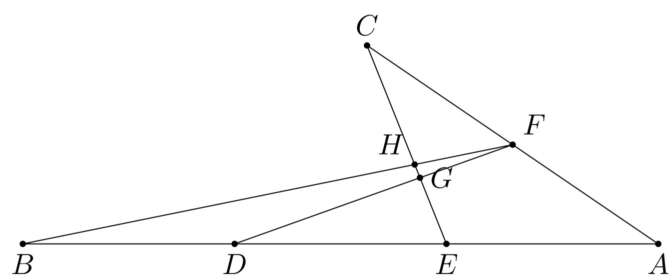
1. (ARML 1996) In $\triangle ABC$, $AB = AC = 115$, $AD = 38$, and $CF = 77$. Compute $\frac{S_{CEF}}{S_{DBE}}$.



2. (ARML 2000) In rectangle $ABCD$, G and H are trisection points of AD, and E and F are trisection points of BC. If $AB = 360$ and $BC = 450$, compute the area of $PQRS$.



3. (ARML 2012) Given noncollinear points A, B, C, segment AB is trisected by points D and E, and F is the midpoint of segment AC. DF and BF intersect CE at G and H, respectively. If $S_{EDG} = 18$, compute S_{FGH}.



4. (ARML 2015) In trapezoid $ABCD$ with bases AB and CD, $AB = 14$ and $CD = 6$. Points E and F lie on AB such that $AD \parallel CE$ and $BC \parallel DF$. Segments DF and CE intersect at G, and AG intersects BC at H. Compute $\frac{S_{CGH}}{S_{ABCD}}$.