Geometry

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ARML Practice 11/04/2012

Warmup

Problem (HMMT 2001 Geometry/1.)

A circle of radius 3 passes through the center of a square of side length 2. Let X be the region inside the circle and outside the square, and Y the region inside the square and outside the circle. What is the positive difference of the areas of X and Y?

Problem (I think this might be a Mathcounts problem maybe?)

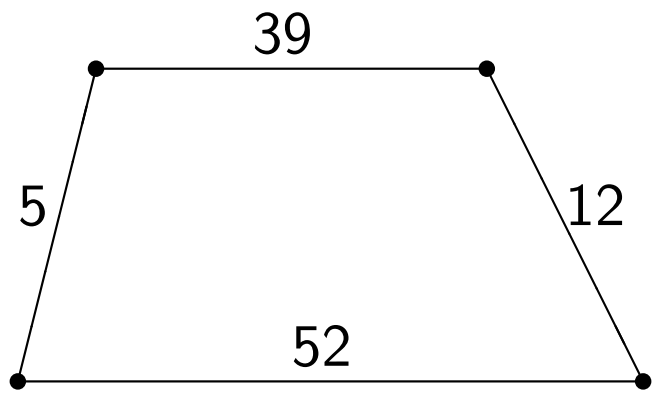
The corner of a square of side length 2 is at the center of another square of side length 2. What is the area of their intersection?

Problem (Well-known result)

Let $\triangle ABC$ be a right triangle with hypotenuse AB. What is the length of the altitude dropped from C onto AB, in terms of the lengths of the sides?

Problem (AMC 12A 2002/25.)

Find the area of the trapezoid below.



Problem (HMMT 2006 Geometry/2.)

Let $\triangle ABC$ be a right triangle. Point P is chosen on the hypotenuse AB such that $\angle ACP = \angle BCP = 45^\circ$, and it turns out that $AP = 1$ and $BP = 2$.

- *What is the ratio of the areas $[ACP] : [BCP]$?*
- *What is the ratio of the lengths $AC : BC$?*
- *What is the area of $\triangle ABC$?*

Problem (Extension of the above)

Now do the same thing if $\angle ACP = 60^\circ$ and $\angle BCP = 30^\circ$.

Problem (Dunno where this is from)

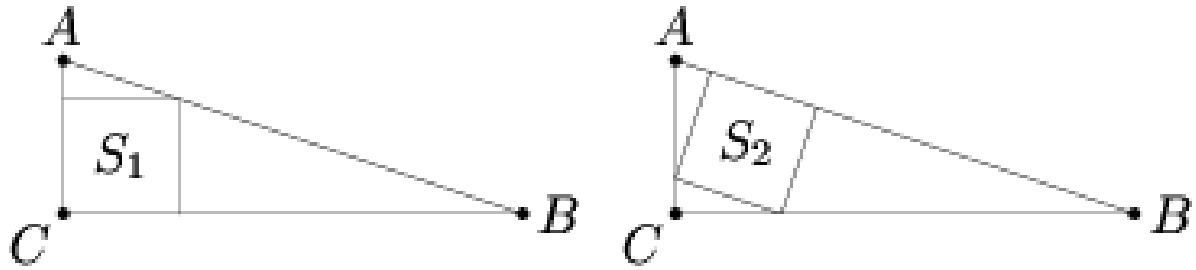
In square $ABCD$, draw the lines from A to the midpoint of BC, from B to the midpoint of CD, from C to the midpoint of AD, and from D to the midpoint of AB. These form a small square inside square $ABCD$. What is the ratio of the areas of the two squares?

Problem (AIME 1985/4.)

Same setup as "Dunno where this is from", except the line from A goes to a point $1/n$ of the way from C to B, and same for the others. The ratio of the areas is $1 : 1985$. What is n?

Problem (AIME 1987/15.)

Let $\triangle ABC$ be a right triangle, with right angle $\angle C$. We inscribe squares in $\triangle ABC$ in two ways:



If the area of S_1 is 441 and the area of S_2 is 440, find $AC + CB$.

Or you could find the area of $\triangle ABC$. Basically just find what you can.