21-268 – Homework assignment week #7

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Reminder

This homework is due next Wednesday before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Exercises (24 pts)

1. (4 pts) Let R be the region in the plane bounded by the curves $x = y^3 - y$, $y = -1$, $y = 1$ and $x = 2$. Compute
$$\iint_R y dA.$$

2. Let $R_1 = \{(x, y) : 0 \leq x \leq 2 \text{ and } \max(1 - x, 3x - 3) \leq y \leq 1 + x\}$ and $R_2 = \{(x, y) : 0 \leq y \leq 3 \text{ and } \max(1 - y, y - 1) \leq x \leq 1 + y/3\}$.
   (a) (2 pts) Prove that if $(x, y) \in R_1$ then $(x, y) \in R_2$.
   (b) (2 pts) Prove that if $(x, y) \in R_2$ then $(x, y) \in R_1$.
   Note: it now follows that $R_1 = R_2$.

3. (3 pts) Evaluate the following integral:
$$\int_0^4 \int_{\sqrt{y}}^2 e^{x^3} dxdy.$$
   Note: there is no closed form antiderivative for e^{x^3}.

4. Let R be the region in the first octant (i.e. the region where $x, y, z \geq 0$) that is bounded by $x = 0$, $y = 0$, $z = x$, and $x^2 + y^2 + z^2 = 4$. Let $f(x, y, z)$ be a continuous function on R.
   (a) (3 pts) Find the limits of integration, A ,B, C, D, E, and F, so that
$$\iiint_R f dV = \int_A^B \int_C^D \int_E^F f(x, y, z) dzdxdy.$$
   Hint: first look at the possible values of y. Once you fix some y there, look at the possible values of x, and once you fix x and y there look at the possible values of z.
   (b) (2 pts) Find the limits of integration, A ,B, C, D, E, and F, so that
$$\iiint_R f dV = \int_A^B \int_C^D \int_E^F f(x, y, z) dydzdx.$$

5. (4 pts) Let R be the region in $\mathbb{E}^3$ that is bounded by the planes $y = 0$, $y = x$, $y = 2 - x$, $z = y$, and $z = 2 - y$. Compute

$$\iiint\limits_R y dV.$$

Hint: do just as in the previous exercice, but pick the order of iterated integrals for which it is easiest to compute the bounds.

6. (4 pts) Compute the integral

$$\int\int_R \frac{x - y}{\sqrt{x^2 + y^2}} dA$$

where R is the triangular region with vertices $(0, 0)$, $(1, 1)$, and $(0, 2)$ (draw it) by converting to polar coordinates.