21-268 – Homework assignment week #6

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Reminder

This homework is due next Friday before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Exercises (23 pts)

1. (5 pts) Consider the function $f(x, y) = x^2 + y^2 - 2x$ on $D = \{(x, y) : 0 \leq x \text{ and } x + y \leq 2 \text{ and } y - x \geq -2\}$. Find the absolute maximum and absolute minimum of f over D, that is find $(x_0, y_0) \in D$ and $(x_1, y_1) \in D$ such that

$$f(x_0, y_0) \leq f(x, y) \leq f(x_1, y_1)$$

for all $(x, y) \in D$.

2. (5 pts) Let $f(x, y) = xy^2$ and $D = \{(x, y) : x^2 + y^2 \leq 4 \text{ and } x \geq 0\}$. Find the maximum and minimum of f over the set D. Find all points where the maximum and minimum values occur.

3. (3 pts) Consider

$$f(x, y) = \frac{x^2 + 2y^2 + 1}{x^4 + y^4 + 1}.$$

Suppose there is $(x_0, y_0) \in \mathbb{E}^2$ such that

$$f(x_0, y_0) \leq f(x, y)$$

for all $(x, y) \in \mathbb{E}^2$. Derive a contradiction from this. *Hint: look at* $f(x, 0)$.

4. (2+1 pts) Let $\vec{h}(x, y, z) = y^2\vec{i} - x\vec{j} + y\vec{k}$. Compute $\vec{\nabla} \times \vec{h}$. Explain in one sentence why there is no scalar function, $f(x, y, z)$ such that $\vec{\nabla} f = \vec{h}$.

5. (4 pts) Assume that $\vec{v}$ and $\vec{w}$ are two differentiable vector fields and prove that

$$\vec{\nabla} \cdot (\vec{v} \times \vec{w}) = \vec{w} \cdot (\vec{\nabla} \times \vec{v}) - \vec{v} \cdot (\vec{\nabla} \times \vec{w}).$$

6. (3 pts) Let R be the disk given by $x^2 + y^2 \leq 1$.

$$\iint_R \frac{3x^2 + 4y^2}{x^2 + y^2} dA$$

is one of the following values. Identify which one: $-\pi/2$, $\pi/2$, $3\pi/2$, $5\pi/2$, $7\pi/2$, $9\pi/2$, $11\pi/2$, $13\pi/2$. Explain briefly but clearly why it must be the one you chose.