21-268 – Homework assignment week #14

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Reminder

Homework is due next Wednesdays before 5pm, to me in class or in Christopher Cox's mailbox in Wean Hall 6113 (pay attention to the arrow !). Late homework will never be accepted without a proper reason. In case of physical absence, electronic submissions by e-mail to both me your TA can be accepted. Please do not forget to write your name, andrew id and section and please use a staple if you have several sheets.

Exercises (22 pts)

1. This problem is an illustration of Kelvin-Stokes theorem. Let

$$\vec{F}(x,y,z) = -y(z+1)\vec{i} + x(z+1)\vec{j}.$$

Let $R = \{(x,y) : x^2 + y^2 \leq 1\}$ and for any real number D let

$$S_D = \left\{(x, y, D(1 - x^2 - y^2) : (x,y) \in R\right\}.$$

(a) (3 pts) What is the boundary of the surface S_D ?

(b) (5 pts) Orienting $C = \left\{(x,y,0) : x^2 + y^2 = 1\right\}$ clockwise, compute (without using Kelvin-Stokes theorem)

$$\int_C \vec{F} \cdot \mathrm{d}\vec{r}.$$

(c) (6 pts) Orienting S_D with a positive $\vec{k}$ component, compute (without using Kelvin-Stokes theorem)

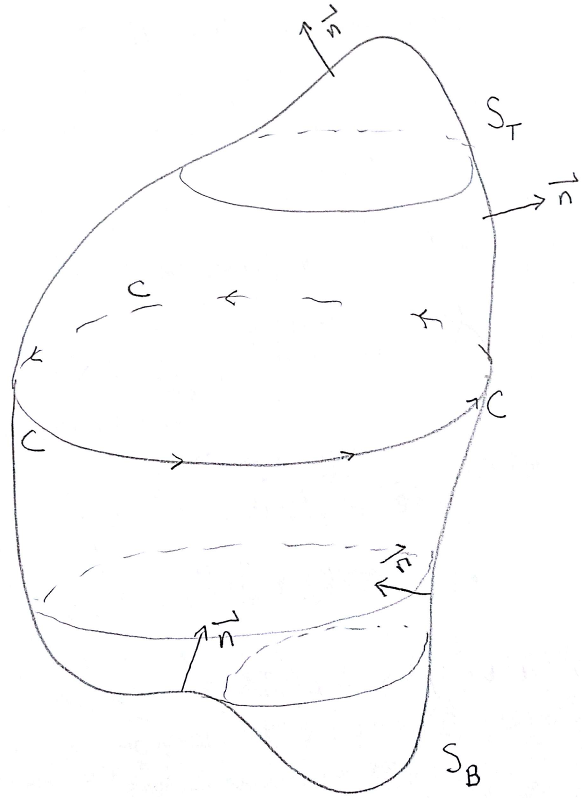
$$\iint_{S_D} \vec{\nabla} \times \vec{F} \cdot \mathrm{d}\vec{\sigma}.$$

2. Let S_T and S_B be two surfaces both bounded by the same curve C, as pictured on the next page. Assume that $S_T \bigcap S_B = C$ and take the normals as pictured on the next page. Let $\vec{F}(x,y,z)$ be a $\mathcal{C}^2$ vector field.

(a) (4 pts pts) Use Kelvin-Stokes theorem to show that

$$\int\int_{S_T} \vec{\nabla} \times \vec{F} \cdot \mathrm{d}\vec{\sigma} = \int\int_{S_B} \vec{\nabla} \times \vec{F} \cdot \mathrm{d}\vec{\sigma}.$$

(b) (4 pts) Use the divergence theorem to show the same result again.


$\vec{n}$
S_T
$\vec{n}$
C
C
C
$\vec{n}$
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S_B