PRACTICE FOR TEST 3, November 19, 2004

MATERIAL COVERED: up to – and including – 3.16

Problem 1: Find the area of the region that lies inside the curve $r = 2|\sin\theta|$ and outside the curve $r = \frac{\sqrt{2}}{2} + |\sin\theta|$.

Problem 2: Exercise 20, page 125.

Problem 3: Assume that $\lim_{x\to p} f(x) = L$ and that $L > 0$. Prove that

$$\lim_{x\to p} (f(x))^{\frac{1}{3}} = L^{\frac{1}{3}}.$$

Problem 4: Assume that $\lim_{x\to p} f(x) = \lim_{x\to p} g(x) = L$ and define $h(x)$ by $h(x) = 2f(x) + g(x)$ if x is rational and $h(x) = f(x) + 2g(x)$ if x is irrational. Prove that

$$\lim_{x\to p} h(x) = 3L.$$

Problem 5: Assume that f is continuous at every point of $[0, 3/2]$ and that $f(x) \geq \frac{1}{1-x}$ for every $x \in (1, 2]$. Show that f is bounded below on $[0, 2]$, that is show that the set $\{f(x) : x \in [0, 2]\}$ is bounded from below.

Problem 6: Exercise 21, page 142.

Problem 7: Exercise 5, page 145.

Problem 8: Prove that
(i) Prove that the equation $x^3 + 2x = x^2 + 1$ has a solution in $(0, 1)$.

(ii) there exists $c \in \mathbb{R}$ such that $f(c) = 10$, where $f(x) := x^3 - x^2 + x$.