ASSIGNMENT 5
Due Thursday, October 14, 2004

Problem 1: Let $a < b$ and let $f(x)$ be defined for all $x \in [a, b]$. Assume there exists a constant L such that $|f(x) - f(y)| \leq L|x - y|$ for all x and y in the interval $[a, b]$ (f is said to be *Lipschitz continuous* on $[a, b]$). Prove that f is integrable on $[a, b]$.

Problem 2: Let $a < b$ and let f and g be bounded functions on $[a, b]$. Prove that

$$\overline{I}(f + g) \leq \overline{I}(f) + \overline{I}(g).$$

Problem 3: Assume that $\sum_{i=1}^{n} \sqrt{i} \leq \frac{2}{3} n\sqrt{n} + \sqrt{n}$ and $\sum_{i=1}^{n-1} \sqrt{i} \geq \frac{2}{3} n\sqrt{n} - \sqrt{2n}$ hold for all positive integers, n. Prove that $\sqrt{x}$ is integrable on [0,1] and that $\int_0^1 \sqrt{x} dx = \frac{2}{3}$.

Problem 4: Exercise 23 on page 83.

Problem 5: Exercise 15 on page 94.