ASSIGNMENT 2
Due Tuesday, September 14, 2004

Problem 1: Do problems 3, 4, 5, 6 on page 28. You may use without proof the result of problem 2 and the fact that a nonempty bounded set of integers has a maximum and a minimum.

Problem 2: Let S_1 and S_2 be nonempty sets of real numbers that are bounded from above. Prove that

$$\sup(S_1 \cup S_2) = \max\{\sup(S_1), \sup(S_2)\}.$$

Note: $S_1 \cup S_2$ is defined to be

$$\{s : s \in S_1 \quad \text{or} \quad s \in S_2\}.$$

Problem 3: Let S be a nonempty set of real numbers that is bounded from below. Define $T := \{-s : s \in S\}$. Show that T is nonempty and bounded from above. Then show that $\sup(T) = -\inf(S)$.

Problem 4: For each positive integer n let a_n and b_n be real numbers. Define the sets $A := \{a_n : \text{n is a positive integer}\}$, $B := \{b_n : \text{n is a positive integer}\}$, and $C := \{a_n + b_n : \text{n is a positive integer}\}$. Show that

$$\sup(C) \leq \sup(A) + \sup(B)$$

and give an example where $\sup(C)$ is strictly less than $\sup(A) + \sup(B)$.

Note: This does not contradict Theorem I.33.