Lecture 22

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1. Find a unit vector that has the same direction as $v = 8i - j + 4k$. $|v| = \sqrt{8^2 + (-1)^2 + 4^2} = \sqrt{81} = 9$, so
$$u = \frac{8}{9}i - \frac{1}{9}j + \frac{4}{9}k$$
is a unit vector with the same direction as v.

2. Find $a \cdot b$ given $|a| = 6,\ |b| = 5$ and the angle between a and b is $2\pi/3$.

   Using the formula
$$a \cdot b = |a||b|\cos(\theta) = 6 \cdot 5\cos(2\pi/3) = 6 \cdot 5(-\frac{1}{2}) = -15$$

3. Find the angle between the vectors $a = \langle 3, -1, 5\rangle$ and $b = \langle -2, 4, 3\rangle$.

   $|a| = \sqrt{3^2 + (-1)^2 + 5^2} = \sqrt{35}$, $|b| = \sqrt{(-2)^2 + 4^2 + 3^2} = \sqrt{29}$, $a \cdot b = 3 \cdot (-2) + (-1) \cdot 4 + 5 \cdot 3 = 5$. So
$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{5}{\sqrt{35}\sqrt{29}} = \frac{5}{\sqrt{1015}}$$
so $\theta = \cos^{-1}(\frac{5}{\sqrt{1015}})$.

4. Find the angle between the vectors $a = 4i - 3j + k$ and $b = 2i - k$.

   $|a| = \sqrt{4^2 + (-3)^2 + 1^2} = \sqrt{26}$, $|b| = \sqrt{2^2 + 0^2 + (-1)^2} = \sqrt{5}$, $a \cdot b = 4 \cdot 2 + (-3) \cdot 0 + 1 \cdot (-1) = 7$. So
$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{7}{\sqrt{26}\sqrt{5}} = \frac{5}{\sqrt{130}}$$
so $\theta = \cos^{-1}(\frac{7}{\sqrt{130}})$.

5. Find the acute angle between the lines $2x - y = 3$ and $3x + y = 7$.

   The slope of the first line is 2 and the slope of the second line is -3. Thus, the direction of the first line can be represented by either $\langle 1, 2\rangle$ or $\langle -1, -2\rangle$. The direction of the second line can be represented by either $\langle 1, -3\rangle$ or $\langle -1, 3\rangle$. Each line has two possible directions because it could go forward or backwards.

   Looking at the lines, it is clear that to obtain an acute angle, we can look at $a = \langle 1, 2\rangle$ and $b = \langle -1, 3\rangle$ as a pair of vectors, because the angle between them is acute. $|a| = \sqrt{1^2 + 2^2} = \sqrt{5}$, $|b| = \sqrt{(-1)^2 + 3^2} = \sqrt{10}$, $a \cdot b = 1 \cdot (-1) + 2 \cdot 3 = 5$. Thus
$$\cos\theta = \frac{a \cdot b}{|a||b|} = \frac{5}{\sqrt{5}\sqrt{10}} = \frac{5}{\sqrt{50}} = \frac{5}{\sqrt{25}} = \frac{\sqrt{2}}{2}$$
so $\theta = \frac{\pi}{4}$.