Lecture 7

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1.

$$\frac{dy}{dt} = \frac{t}{ye^{y+t^2}}$$
$$\int ye^y dy = \int te^{-t^2} dt$$

Using integration by parts

$$\int ye^y dy = ye^y - \int e^y dy = ye^y - e^y + C_1$$

Clearly

$$\int te^{-t^2} dt = -\frac{1}{2}e^{-t^2} + C_2$$

Thus a solution is

$$(y-1)e^y = -\frac{1}{2}e^{-t^2} + C$$

2.

$$\frac{dp}{dt} = t^2p - p + t^2 - 1$$
$$\frac{dp}{dt} = (t^2-1)(p+1)$$
$$\int \frac{dp}{p+1} = \int (t^2-1)dt$$
$$\ln|p+1| + C_1 = \frac{1}{3}t^3 - t + C_2$$
$$p + 1 = e^{\frac{1}{3}t^3 - t} + C$$
$$p = Ke^{\frac{1}{3}t^3 - t} - 1$$

by letting $K = e^C$.

3. Given $u(0) = -5$ and

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$$
$$\int 2udu = \int 2t + \sec^2 t dt$$
$$u^2 + C_1 = t^2 + \tan t + C_2$$
$$u = \pm\sqrt{t^2 + \tan t + C}$$

then using $u(0) = \pm\sqrt{0^2 + 0 + C} = -5$ gives

$$u = -\sqrt{t^2 + \tan t + 25}$$

4. Given $y(\pi/3) = a$ and $0 < x < \frac{\pi}{2}$ and

$$\begin{aligned} y' \tan x &= a + y \\ \frac{dy}{dx} \tan x &= a + y \\ \int \frac{dy}{a+y} &= \int \cot(x) dx \\ \ln|a+y| + C_1 &= \ln|\sin x| + C_2 \\ a + y &= e^{\ln(\sin x) + C} \\ y &= K \sin x - a \end{aligned}$$

noting that since $0 < x < \frac{\pi}{2}$ so $\sin x$ is positive.

Now using $y(\pi/3) = K \sin \frac{\pi}{3} - a = a$ gives $K\frac{\sqrt{3}}{2} - a = a$ so $K = \frac{4a}{\sqrt{3}}$ thus the solution is

$$y = \frac{4a}{\sqrt{3}} \sin x - a$$

5. Find an equation of the curve that passes through the point $(0, 1)$ and whose slope at (x, y) is xy.

We want $\frac{dy}{dx} = xy$ with $y(0) = 1$. To solve,

$$\begin{aligned} \frac{dy}{dx} &= xy \\ \int \frac{dy}{y} &= \int x dx \\ \ln|y| + C_1 &= \frac{x^2}{2} + C_2 \\ y &= Ke^{x^2/2} \end{aligned}$$

then using $y(0) = Ke^{0^2/2} = 1$ so $K = 1$ so the solution is

$$y = e^{x^2/2}$$