Lecture 5

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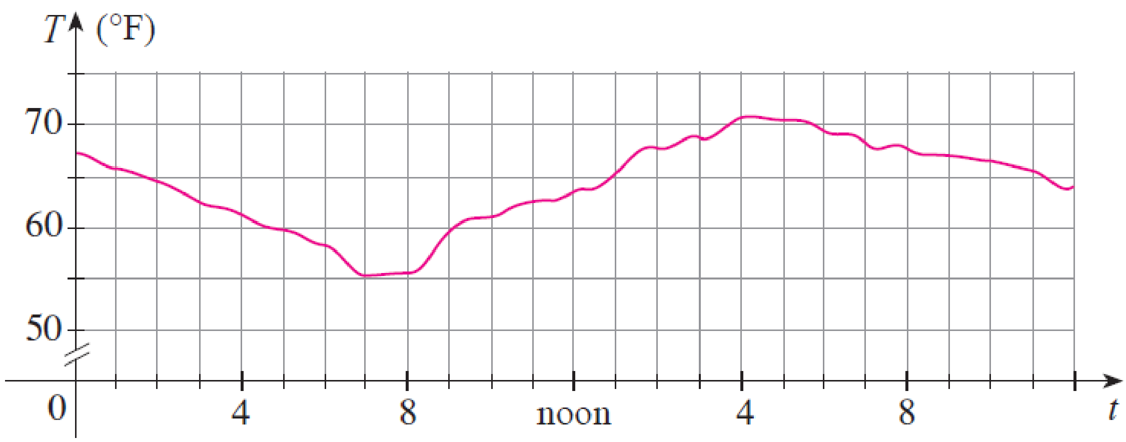
February 3, 2014

1.

33. A graph of the temperature in New York City on September 19, 2009 is shown. Use Simpson's Rule with $n = 12$ to estimate the average temperature on that day.



Recall estimation using Simpson's Rule

$$\int_a^b f(x)dx \approx S_n = \frac{\Delta x}{3}[f(x_0) + 4f(x_1) + 2f(x_2) + 4f(x_3) + \cdots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)]$$

using $n = 12$, we obtain the values

x	$f(x)$
0	67
2	65
4	61
6	58
8	56
10	61
12	63
14	68
16	71
18	69
20	67
22	66
24	64

$$S_{12} = \frac{2}{3}[67 + 4 \cdot 65 + 2 \cdot 61 + 4 \cdot 58 + 2 \cdot 56 + 4 \cdot 61 + 2 \cdot 63 + 4 \cdot 68 + 2 \cdot 71 + 4 \cdot 69 + 2 \cdot 67 + 4 \cdot 66 + 64]$$

which gives $S_{12} \approx 1543.33$. Therefore, the average value is

$$\frac{1587.33}{24} \approx 64.31$$

2. The region bounded by the curves $y = e^{-1/x}$, $y = 0, x = 1$, and $x = 5$ is rotated around the x-axis. Use Simpson's Rule with $n = 8$ to estimate the volume of the resulting solid.

We wish to approximate

$$\int_1^5 \pi \cdot (e^{-1/x})^2 dx$$

x	$\pi \cdot (e^{-1/x})^2$
1.0	0.425
1.5	0.828
2.0	1.156
2.5	1.412
3.0	1.613
3.5	1.774
4.0	1.905
4.5	2.014
5.0	2.106

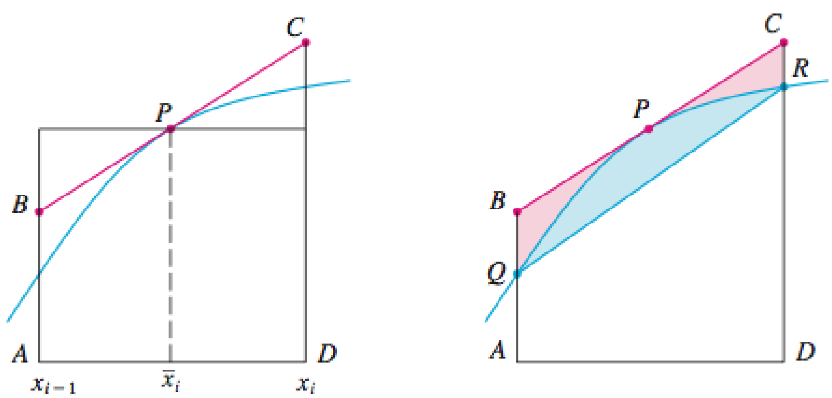
$$S_8 = \frac{0.5}{3}[0.425 + 4 \cdot 0.828 + 2 \cdot 1.156 + 4 \cdot 1.412 + 2 \cdot 1.618 + 4 \cdot 1.774 + 2 \cdot 1.905 + 4 \cdot 2.014 + 2.106]$$

which gives $S_8 = 5.999$.

3. Sketch the graph of a continuous function on $[0, 2]$ for which the Trapezoidal Rule with $n = 2$ is more accurate than the Midpoint Rule.

4. If f is a positive function and $f''(x) < 0$ for $a \leq x \leq b$, show that $T_n < \int_a^b f(x)dx < M_n$.

If $f''(x) < 0$, then f is concave down. See Figure 5 on p.533.



The line $\overline{BC}$ is chosen to be tangent to point P on the curve.