Lecture 3

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1.

$$\int_0^1 \frac{2}{2x^2+3x+1}dx = \int_0^1 \frac{4}{2x+1} + \frac{-2}{x+1}dx = 2\ln|2x+1| - 2\ln|x+1| + C\Big]_0^1 = 2\ln 3 - 2\ln 2 = \ln\frac{9}{4}$$

Using the partial fraction

$$\frac{2}{(2x+1)(x+1)} = \frac{A}{2x+1} + \frac{B}{x+1}$$

$$2 = A(x+1) + B(2x+1) = (A+2B)x + (A+B)$$

so $A = 4, B = -2$.

2.

$$\int_3^4 \frac{x^3-2x^2-4}{x^3-2x^2}dx = \int_3^4 1 + \frac{1}{x} + \frac{2}{x^2} - \frac{1}{x-2}dx = x + \ln|x| - \frac{2}{x} - \ln|x-2|\Big]_3^4$$

$$= (4-3) + (\ln 4 - \ln 3) - (\frac{2}{4} - \frac{2}{3}) - (\ln 2 - \ln 1) = 1 + \ln\frac{4}{3} + \frac{1}{6} - \ln 2$$

Using the partial fraction

$$\frac{x^3-2x^2-4}{x^3-2x^2} = 1 + \frac{-4}{x^2(x-2)} = 1 + \frac{A}{x} + \frac{B}{x^2} + \frac{C}{x-2}$$

$$-4 = A(x-2)x + B(x-2) + Cx^2$$

so $A + C = 0, -2A + B = 0, -2B = -4$, thus $B = 2, A = 1, C = -1$.

3.

$$\int \frac{10}{(x-1)(x^2+9)}dx = \int \frac{1}{x-1} + \frac{-x-1}{x^2+9}dx = \int \frac{1}{x-1} + \frac{-x}{x^2+9} + \frac{-1}{x^2+9}dx$$

$$= \ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\tan^{-1}(\frac{x}{3}) + C$$

note $\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}(\frac{x}{a}) + C$.

Using the partial fraction

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+9}$$

$$10 = A(x^2+9) + (Bx+C)(x-1) = (A+B)x^2 + (C-B)x + (9A-C)$$

thus $-A = B = C$ and $10 = 9A - C$ so $A = 1, B = C = -1$.

4.

$$\int \frac{4x}{x^3+x^2+x+1}dx = \int \frac{-2}{x+1} + \frac{2x+2}{x^2+1}dx = -2\ln|x+1| + \ln|x^2+1| + 2\tan^{-1}(x) + C$$

Using partial fraction

$$\frac{4x}{x^3+x^2+x+1} = \frac{4x}{(x+1)(x^2+1)} = \frac{A}{x+1} + \frac{Bx+C}{x^2+1}$$

$$4x = A(x^2+1) + (Bx+C)(x+1) = (A+B)x^2 + (B+C)x + (A+C)$$

so $-A = B = C$ and $B + C = 4$ so $B = C = 2$ and $A = -2$.