Lecture 1

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Review the following formulas

1.
$$\int \tan(x)dx = -\ln|\cos(x)| + C$$

We perform the substitution $u = \cos(x)$ then $du = -\sin(x)dx$, giving

$$\int \tan(x)dx = \int \frac{\sin(x)}{u}\frac{du}{-\sin(x)} = \int -\frac{1}{u}du = -\ln|u| + C = -\ln|\cos(x)| + C$$

2.
$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

We perform the substitution $u = \sec(x) + \tan(x)$ so $du = (\sec(x)\tan(x) + \sec^2(x))dx$

$$\begin{aligned}\int \sec(x)dx &= \int \sec(x)\frac{\sec(x)+\tan(x)}{\sec(x)+\tan(x)}dx = \int \frac{\sec^2(x)+\sec(x)\tan(x)}{\sec(x)+\tan(x)}\frac{du}{\tan(x)+\sec^2(x)} \\ &= \int \frac{1}{u}du = \ln|u| + C = \ln|\sec(x)+\tan(x)| + C\end{aligned}$$

Now do the following problems

1.
$$\int (2x^3 - 4x + 3)dx = 2\frac{x^4}{4} - 4\frac{x^2}{2} + 3x + C = \frac{1}{2}x^4 - 2x^2 + 3x + C$$

2. Integration by parts

Recall $uv = \int udv + vdu$ so $\int udv = uv - \int vdu$. Let $u = x$ and $dv = \sin(x)dx$ so $dv = 1dx$ and $v = -\cos(x)$. Thus,

$$\int x\sin(x)dx = -x\cos(x) + \int \cos(x)dx = -x\cos(x) + \sin(x) + C$$

3. Using long division, we obtain

$$\int \frac{x}{3x+4}dx = \int \frac{1}{3} - \frac{4}{3(3x+4)}dx = \frac{1}{3}x - \frac{4}{9}\ln|3x+4| + C$$

4. We make the substitution $u = x^2 - 2x$ so $du = (2x-2)dx$, and $w = \frac{x+1}{\sqrt{7}}$ so $\sqrt{7}dw = dx$, so

$$\begin{aligned}\int \frac{x+3}{x^2-2x+8}dx &= \int \frac{\frac{1}{2}(2x-2)+4}{x^2+2x+8}dx = \int \frac{\frac{1}{2}}{u+8}du + \int \frac{4}{x^2+2x+8}dx \\ &= \frac{1}{2}\ln|u+8| + \int \frac{4}{x^2+2x+8}dx = \frac{1}{2}\ln|u+8| + \int \frac{4}{(x+1)^2+7}dx \\ &= \frac{1}{2}\ln|u+8| + \int \frac{4}{7(w^2+1)}\sqrt{7}dw \\ &= \frac{1}{2}\ln|u+8| + \frac{4}{7}\sqrt{7}\tan^{-1}(w) \\ &= \frac{1}{2}\ln|x^2+2x+8| + \frac{4}{\sqrt{7}}\tan^{-1}(\frac{x-1}{\sqrt{7}})\end{aligned}$$

5. Recall the double angle formula $\cos(2x) = 1 - 2\sin^2(x)$

$$\int (\sin(x))^2 dx = \int \frac{1}{2}(1 - \cos(2x))dx = \frac{1}{2}x - \frac{1}{4}\sin(2x) + C$$

Review derivatives

$$\frac{d}{dx}\ln(x) = \frac{1}{x} \qquad \frac{d}{dx}e^x \qquad \frac{d}{dx}\tan^{-1}(x) = \frac{1}{\sec^2(\tan^{-1}(x))} = \frac{1}{1+\tan^2(\tan^{-1}(x))} = \frac{1}{1+x^2}$$

$$\frac{d}{dx}\sin^{-1}(x) = \frac{1}{\cos(\sin^{-1}x)} = \frac{1}{\sqrt{1-\sin^2(\sin^{-1}(x))}} = \frac{1}{\sqrt{1-x^2}}$$