MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#7

GSI: CHRISTOPHER EUR, DATE: 10/13/2017

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Problem 1. Consider the matrix

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

(a) (2 points) Determine whether A is invertible by computing $\det A$.
(b) (3 points) For each eigenvalue of A, find a basis for its eigenspace.

Problem 2. (5 points) Let $\mathbb{P}_2 := \{a_0 + a_1 t + a_2 t^2 : a_0, a_1, a_2 \in \mathbb{R}\}$ be the vector space of polynomials of degree ≤ 2. Consider the linear map

$$T : \mathbb{P}_2 \to \mathbb{P}_2 \text{ defined by } p(t) \mapsto p(t) + p'(t)$$

(a) (3 points) Letting $B = \{1, t, t^2\}$ be a basis for $\mathbb{P}_2$, write down the matrix of the linear transformation ${}_B[T]_B$.
(b) (2 points) Find all polynomials $p(t) \in \mathbb{P}_2$ such that $T(p(t)) = p(t)$.

#1. (a) Upper $\Delta \Rightarrow \quad 1 \cdot 1 \cdot 1 = 1 \neq 0$. $\boxed{\det A = 1 \neq 0} \Rightarrow$ A invtbl.

(b) Upper $\Delta \Rightarrow$ 1 the only eigenval. $\Rightarrow \operatorname{nul}(A - I) = \operatorname{nul}\left(\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix}\right) = \operatorname{span}\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\}$.

Since $\operatorname{rk}(A - I) = 2$, $\operatorname{nullity}(A - I) = 1$, so all good. $\therefore \boxed{\left\{\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}\right\} \text{ for } \lambda = 1}$

#2. (a) $T(1) = 1$, $T(t) = t + 1$, $T(t^2) = t^2 + 2t$

$\Rightarrow \quad {}_B[T]_B = \boxed{\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}}$.

(b) Equiv. to finding eigenspace for T w/ $\lambda = 1$.
Well, #1(b) says $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}_B$ is the basis for it.

$\therefore \boxed{c \in \mathbb{P}_2 \text{ for any } c \in \mathbb{R}}$