MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#3

GSI: CHRISTOPHER EUR, DATE: 9/15/2017

STUDENT NAME: Midterm Season

Problem 1. (6 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that

$$T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix} \quad \text{and} \quad T\left(\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix}.$$

(a): Write down the values of Te_1 and Te_2 where e_1, e_2 are standard vectors of $\mathbb{R}^2$. Use this to write down the matrix associated to T.

(b): *Without referring to the matrix of* T, explain why T is not one-to-one.

Problem 2. (4 points) Let A be a $m \times n$ matrix, and suppose that there exist a matrix $B_{n\times m}$ such that $BA = \mathrm{Id}_n$. Show that A then has linearly independent columns.

#1. (a) $Te_2 = T\left(\begin{bmatrix}2\\3\end{bmatrix} - 2\begin{bmatrix}1\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}2\\2\end{bmatrix} = \begin{bmatrix}-1\\-1\end{bmatrix}$

$Te_1 = T\left(\begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\1\end{bmatrix} - \begin{bmatrix}-1\\-1\end{bmatrix} = \begin{bmatrix}2\\2\end{bmatrix}.$

$\Rightarrow$ matrix $= \begin{bmatrix}2 & -1\\2 & -1\end{bmatrix}$

(b) $T\left(\begin{bmatrix}2\\3\end{bmatrix}\right) = T\left(\begin{bmatrix}1\\1\end{bmatrix}\right)$ but $\begin{bmatrix}2\\3\end{bmatrix} \neq \begin{bmatrix}1\\1\end{bmatrix}$.

#2. Suppose $A\vec{x} = \vec{0}$. Then $\vec{0} = BA\vec{x} = \mathrm{Id}_n \vec{x} = \vec{x}$.
This shows that $A\vec{x} = \vec{0}$ only has the trivial soln.
Hence, col. of A are lin. indep.