MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#13

GSI: CHRISTOPHER EUR, DATE: 12/1/2017

STUDENT NAME: Taxation w/o representation

Problem 1. (5 points) Solve for $u(x,t)$ where $-\pi < x < \pi$ and $t > 0$ satisfying the following (wave) equation:

$$4u_{xx} = u_{tt}$$
$$u(-\pi,t) = u(\pi,t) = 0 \quad \forall t > 0$$
$$u(x,0) = \sin 3x + 4\sin 5x$$
$$u_t(x,0) = \cancel{2\cos 4x} \quad 2\sin 4x$$

Problem 2. (5 points) Solve for $u(x,t)$ where $-\frac{\pi}{2} < x < \frac{\pi}{2}$ and $t > 0$ satisfying the following (heat) equation:

$$5u_{xx} = u_t$$
$$u(-\tfrac{\pi}{2},t) = u(\tfrac{\pi}{2},t), \quad u_x(-\tfrac{\pi}{2},t) = u_x(\tfrac{\pi}{2},t) \quad \forall t > 0 \text{ (CAREFUL HERE!)}$$
$$u(x,0) = 3\sin 2x + 5\cos 6x$$

#1. $u_m = \sin mx\,(\sin 2mt + \cos 2mt)$ (no $\cos mx$ since we need 0 at $x=-\pi$ and $x=\pi$).

since $4\cdot m^2 = (2m)^2$ from $\frac{\partial^2}{\partial t^2}$

$4\cdot\frac{\partial^2}{\partial x^2}$ from

Now, $u(x,0) = \sin 3x + 4\sin 5x \Rightarrow u = \sin 3x \cos 6t + 4\sin 5x \cos 10t + \sum_m \sin mx \sin 2m$

$u_t(x,0) = 2\cos 4x \Rightarrow \sin 4x \cancel{\cos}\ \sin 8t$ term has coeff. $2/8$

$$\boxed{\therefore\ u(x,t) = \sin 3x\cos 6t + 4\sin 5x \cos 10t + \left(\frac{1}{\cancel{2}\,4}\right)\sin 4x \sin 8t}$$

#2. $u_m = \sin 2mx\ e^{-5\cdot 4m^2 t}$ or $\cos 2mx\ e^{-5\cdot 4m^2 t}$

$\sin 2mx$, $\cos 2mx$ both have period $\frac{2\pi}{2m} = \frac{\pi}{m}$, so the boundary condition is satisfied.

Now, $u(x,0) = 3\sin 2x + 5\cos 6x \Rightarrow u = 3\sin 2x\, e^{-5\cdot 4t} + 5\cos 6x\, e^{-5\cdot 4\cdot 9t}$

$$\boxed{\therefore\ u(x,t) = 3\sin 2x\, e^{-20t} + 5\cos 6x\, e^{-180t}}$$