MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#12

GSI: CHRISTOPHER EUR, DATE: 11/17/2017

STUDENT NAME: ______________________

Problem 1. Consider the homogeneous differential equation

$$y'' - 4y' + 4y = 0$$

(a) (3 points) Find the general solution as the span of two functions.
(b) (2 points) Confirm that the two functions you found in part (a) are linearly independent by computing the Wronskian.

Problem 2. (5 points) Find the general solution to the differential equation

$$y'' + 9y = 6\sin 3t$$

#1. (a) Aux. eq. : $r^2 - 4r + 4 = (r-2)^2 \Rightarrow \boxed{\operatorname{span}_{\mathbb{R}}(e^{2x},\ xe^{2x})}$

(b) $\begin{vmatrix} e^{2x} & xe^{2x} \\ 2e^{2x} & e^{2x} + 2xe^{2x} \end{vmatrix} = e^{4x} + \cancel{2x}e^{4x} - \cancel{2x}e^{4x} = e^{4x} \neq 0 \quad \forall x \in \mathbb{R}.$

Hence $\{e^{2x}, xe^{2x}\}$ lin. indep.

#2. Homog part: $r^2 + 9 = 0 \Rightarrow r = \pm 3i$

Let's solve $y'' + 9y = 6(\cos 3t + i \sin 3t) = 6e^{3it}$ and take the imaginary part.

Guess: Ate^{3it} : $(te^{3it})' = e^{3it} + 3ite^{3it}$

$(te^{3it})'' = 3ie^{3it} - 9te^{3it} + 3i\cancel{t}e^{3it}$

So, $(Ate^{3it})'' + 9(Ate^{3it}) = 6iAe^{3it} = 6e^{3it}$

$\therefore A = -i \Rightarrow$ soln : $-ite^{3it} = -it\cos 3t + t\sin 3t$

$\therefore -t\cos 3t$ is particular soln.

$\therefore$ General soln : $\boxed{-t\cos 3t + \operatorname{span}_{\mathbb{R}}(\cos 3t,\ \sin 3t)}$