MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#10

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Problem 1. Let $A = \begin{bmatrix} 3 & 6 & 3 \\ 4 & 8 & 4 \\ 0 & 0 & 1 \end{bmatrix}$.

(a) (3 points) Find an orthonormal basis for column space of A.

(b) (3 pionts) Find the least-squares solution to $A\vec{x} = \vec{b}$ where $\vec{b} = \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$.

Problem 2. (4 points) True/False: Consider a pairing on $\mathbb{P}_2$ defined as follows:

$$\langle p(t), q(t) \rangle := \int_0^1 p(t) + q(t)dt$$

This is an inner-product on $\mathbb{P}_2$.

#1. (a) Call the columns a_1, a_2, a_3. Note $a_2 = 2a_1$, so $\mathrm{Col}\, A = \mathrm{span}\,(a_1, a_3)$.

Now run Gram-Schmidt: $u_1 = \frac{a_1}{\|a_1\|} = \frac{a_1}{\sqrt{9+16}} = \frac{a_1}{5} = \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}$

$\|u_2\|\, u_2 = a_3 - \langle a_3, u_1 \rangle u_1 = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} - \left(\frac{9}{5} + \frac{16}{5}\right)\begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix} - \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

$\therefore$ orthonormal basis : $\left\{ \begin{bmatrix} 3/5 \\ 4/5 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\} = \{u_1, u_2\}$

(b) $\mathrm{proj}_{\mathrm{Col}A}\, \vec{b} = (b \cdot u_1)u_1 + (b \cdot u_2)u_2 = \left(\frac{21+4}{5}\right) u_1 + 1\, u_2$

$= \begin{bmatrix} 3 \\ 4 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$ (u_1, u_2 from part (a)).

Soln. to $Ax = \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$: particular soln : $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$. ker $A = \begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ $\Rightarrow \left\{ \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + s\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix} : s \in \mathbb{R} \right\}$

#2. False : not positive-definite.

Witness: $p(t) = -1 = q(t)$.

Then $\langle p(t), p(t) \rangle = \int_0^1 -2\, dt = -2 < 0$.