

MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#9

GSI: CHRISTOPHER EUR, DATE: 10/28/2016

STUDENT NAME: _____

Problem 1. Define a linear operator $L : \mathcal{P}_2 \rightarrow \mathcal{P}_2$ by $L(p(x)) = p''(x) - 2xp'(x)$ (where \mathcal{P}_2 is the vector space of polynomials with real coefficients of degree ≤ 2).

- (a) (2 points) Write down the matrix A that represents this linear operator L with respect to the basis $E = (1, x, x^2)$ on \mathcal{P}_2 .
- (b) (3 points) Find all eigenvalues of L and find the basis for each corresponding eigenspaces.
- (c) (2 points) Use the previous part to find a matrix P such that $P^{-1}AP$ is diagonal, and check that $P^{-1}AP$ is indeed diagonal.
- (d) (2 points) Compute $L^{50}(x^2)$ (You need not compute out powers of a single number such as 5^{23}).
- (e) (1 point) Show that there is no polynomial $q(x)$ of degree ≤ 2 such that $L(q(x)) = 5q(x)$.