# MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#7

GSI: CHRISTOPHER EUR, DATE: 10/14/2016

STUDENT NAME: ________________________________

*Problem 1.* (4 points) Suppose $A\vec{x} = \vec{b}$ is an *inconsistent* system of equations. Then show that

$$\operatorname{rank} A < \operatorname{rank}[A|\vec{b}]$$

where $[A|\vec{b}]$ is the matrix $A$ augmented by a column $\vec{b}$.

*Problem 2.* Let $\mathcal{P}_2 := \{a_2x^2 + a_1x + a_0 : a_2, a_1, a_0 \in \mathbb{R}\}$ be the vector space of polynomials of degree at most 2. Let $B = (x^2, x, 1)$ be a basis of $\mathcal{P}_2$. Consider a linear map $T : \mathcal{P}_2 \to \mathbb{R}$ given by

$$T(p(x)) = \int_0^1 p(t)dt$$

(a) (2 points) Write down the matrix $A$ of the linear map $T$ with respect to the basis $B$ on $\mathcal{P}_2$ and the standard basis on $\mathbb{R}$.
(b) (1 point) Verify that $\int_0^1 x^2 + 2x = \frac{4}{3}$ by multiplying $A$ to the coordinate vector corresponding to $x^2 + 2x$ (w/r/t basis $B$) .
(c) (2 points) Find a basis for $\ker T$, and a basis for $\operatorname{im} T$.
(d) (1 point) Verify the rank-nullity theorem for this linear map $T$.