MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#2

GSI: CHRISTOPHER EUR, DATE: 9/9/2016

STUDENT NAME: Bad Gauss

Problem 1. (5 points) Suppose A is a 3×5 matrix and $\vec{u}_1, \vec{u}_2 \in \mathbb{R}^5$ such that $A\vec{u}_1 = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}$ and $A\vec{u}_2 = \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$. Find ~~the~~ a solution for $A\vec{x} = \begin{bmatrix} -4 \\ 6 \\ 1 \end{bmatrix}$ in terms of $\vec{u}_1$ and $\vec{u}_2$.

$$\begin{bmatrix} -4 \\ 6 \\ 1 \end{bmatrix} \overset{?}{\in} \text{span}\left(\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} \right) \qquad \left[\begin{array}{cc|c} 1 & 2 & -4 \\ 3 & 0 & 6 \\ 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 1 & 2 & -4 \\ 2 & 1 & 1 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 2 & -6 \\ 0 & 1 & -3 \end{array}\right] \sim \left[\begin{array}{cc|c} 1 & 0 & 2 \\ 0 & 1 & -3 \\ 0 & 0 & 0 \end{array}\right]$$

Thus, $2\begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} - 3\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -4 \\ 6 \\ 1 \end{bmatrix}$. Hence, $A(2\vec{u}_1 - 3\vec{u}_2)$

$$= 2(A\vec{u}_1) - 3(A\vec{u}_2) = \begin{bmatrix} -4 \\ 6 \\ 1 \end{bmatrix}$$

$\therefore$ Soln: $\boxed{2\vec{u}_1 - 3\vec{u}_2}$

Problem 2. (5 points) Let $v_1, v_2, v_3 \in \mathbb{R}^3$ such that $\{v_1, v_2\}$, $\{v_2, v_3\}$, and $\{v_3, v_1\}$ are each linearly independent sets of vectors. Then is it necessarily true that $\{v_1, v_2, v_3\}$ is a linearly independent set of vectors? If true, prove it. If false, give a counterexample.

$\boxed{\text{No}}$ E.g. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$

$v_1 + v_2 = v_3 \rightarrow$ clearly not lin. indep.

but $\{v_1, v_2\} \rightarrow$ lin. indep. $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\{v_2, v_3\} \rightarrow$ lin. indep. $\begin{bmatrix} 0 & 1 \\ 1 & 1 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

$\{v_3, v_1\} \rightarrow$ lin. indep. $\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & 1 \\ 1 & 0 \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$

Remark Thinking geometrically, I need three vectors that lie on a plane, and no two are scalars of each other.