MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#11-2

GSI: CHRISTOPHER EUR, DATE: 11/18/2016

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Note. You may use a calculator or Wolfram|Alpha to compute definite integrals. (However, you'll need to review how to integrate certain functions anyway, so might as well review it now).

Problem 1. Define an inner product on $C^\infty[-\pi,\pi]$ (the space of all infinitely differentiable functions on the interval $[-\pi,\pi]$) as follows:

$$\langle f(t), g(t)\rangle := \int_{-\pi}^{\pi} f(t)g(t)dt$$

(a) (1 point) Check that $(\sin t, \cos t)$ is an orthogonal set of vectors in $C^\infty[-\pi,\pi]$ with respect to this inner product.

(b) (4 points) Let $W := \mathrm{span}_{\mathbb{R}}(\sin t, \cos t)$ be a subspace of $C^\infty[-\pi,\pi]$, and define $\ell(y) := y''$. Find the function $f(t) \in W$ that "best solves" the equation $\ell(y) = t$; more precisely, find the function $f(t) \in W$ that minimizes

$$\int_{-\pi}^{\pi} \left(t - \ell(f(t))\right)^2 dt$$

Problem 2. Let A and B be orthogonally diagonalizable $n \times n$ matrices.

(a) (2 points) Show that A and B are symmetric.

(b) (3 points) Show that if $AB = BA$, then AB is also orthogonally diagonalizable.

#1. (a) $\int_{-\pi}^{\pi} (\sin t)(\cos t)\, dt = \int_{-\pi}^{\pi} \frac{1}{2}\sin(2t)\, dt = -\frac{1}{4}\cos(2t)\Big]_{-\pi}^{\pi} = 0.$ ✓

(b) Need find $\mathrm{Proj}_W\, t = \dfrac{\langle t, \sin t\rangle}{\langle \sin t, \sin t\rangle}\sin t + \underset{\substack{\| \\ 0}}{\dfrac{\langle t, \cos t\rangle}{\langle \cos t, \cos t\rangle}}\cos t$

0 since t is odd, $\cos t$ even.

$= \dfrac{\int_{-\pi}^{\pi} t\sin t\, dt}{\pi}\sin t = \dfrac{[\sin t - t\cos t]_{-\pi}^{\pi}}{\pi}\sin t = 2\sin t$

Now, want $\ell(f) = 2\sin t$. $f'' = 2\sin t \Rightarrow \boxed{f(t) = -2\sin t}$

#2 (a) $A = PDP^{-1}$ for some D diagonal & $P^{-1} = P^T$.

$A^T = (P^T)^T D^T P^T = PDP^T = A$. Hence A symmetric. Same for B. ✓

(b) $(AB)^T = B^T A^T \underset{\substack{\uparrow \\ \text{from part (a)}}}{=} BA = AB \Rightarrow AB$ symmetric $\Rightarrow$ ortho. diagonalizable. ✓