MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#11-2

GSI: CHRISTOPHER EUR, DATE: 11/18/2016

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Note. You may use a calculator or Wolfram|Alpha to compute definite integrals. (However, you'll need to review how to integrate certain functions anyway, so might as well review it now).

Problem 1. Define an inner product on $C^\infty[-\pi, \pi]$ (the space of all infinitely differentiable functions on the interval $[-\pi, \pi]$) as follows:

$$\langle f(t), g(t) \rangle := \int_{-\pi}^{\pi} f(t)g(t)dt$$

(a) (1 point) Check that $(\sin t, \cos t)$ is an orthogonal set of vectors in $C^\infty[-\pi, \pi]$ with respect to this inner product.

(b) (4 points) Let $W := \text{span}_{\mathbb{R}}(\sin t, \cos t)$ be a subspace of $C^\infty[-\pi, \pi]$, and define $\ell(y) := y''$. Find the function $f(t) \in W$ that "best solves" the equation $\ell(y) = t$; more precisely, find the function $f(t) \in W$ that minimizes

$$\int_{-\pi}^{\pi} \Big(t - \ell(f(t))\Big)^2 dt$$

Problem 2. Let A and B be orthogonally diagonalizable $n \times n$ matrices.

(a) (2 points) Show that A and B are symmetric.

(b) (3 points) Show that if $AB = BA$, then AB is also orthogonally diagonalizable.