MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#6

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Problem 1. Let $\langle \cdot, \cdot \rangle$ be an inner product on $\mathbb{P}_2$ given by

$$\langle p(x), q(x) \rangle = p(-1)q(-1) + p(0)q(0) + p(1)q(1).$$

Compute the projection of $f(x) = 3 + 2t^2$ onto the subspace spanned by $g(x) = 3t - t^2$.

Problem 2. Let $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$. Given that $v_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $v_2 = \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ are eigenvectors of A, find another eigenvector $v_3 \in \mathbb{R}^3$ of A such that $\{v_1, v_2, v_3\}$ is a basis of $\mathbb{R}^3$.

(#1) $$\frac{\langle f,g\rangle}{\langle g,g\rangle} g = \frac{5(-4) + 3(0) + 5(2)}{(-4)(-4) + (0)(0) + (2)(2)} (3t - t^2)$$

$$= \frac{-10}{20} = \boxed{-\frac{1}{2}(3t - t^2)}$$

(#2) A is symmetric $\Rightarrow$ $\exists$ orthogonal eigenbasis.

Since v_1, v_2 eigenvec. & orthogonal $(v_1 \cdot v_2 = -1 + 0 + 1 = 0)$

need find v_3 orthogonal to both v_1, v_2.

$$\ker \begin{bmatrix} 1 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix} = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \right\}. \quad \boxed{v_3 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}.}$$

(Can check $A v_3 = 1 v_3$).

Alternatively, since the problem did not impose orthogonal eigenbasis, any vector v_3 st $Av_3 = v_3$ and $\{v_2, v_3\}$ lin. indep will do.