MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#3

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Problem 1. Assume that the matrix A is row equivalent to B. What is $\operatorname{rank} A$ and $\dim \operatorname{Nul} A$?. Then find bases for $\operatorname{Col} A$, and $\operatorname{Nul} A$.

$1 \quad 0 \quad -2 \quad 4 \quad 5 \quad -6 \rightsquigarrow$ for REF

$$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 0 & 1 & -1 & 3 & 4 & -3 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Problem 2. Let V and W be vector spaces, and let $T : V \to W$ be a linear transformation. Given a subspace U of V, let $T(U)$ denote the set of all images of the form $T(x)$, where $x \in U$. Show that $T(U)$ is a subspace of W .

(1) 2 pivots, 4 free col. $\Rightarrow$ rank $A = 2$, dim Nul $A = 4$.

Basis of $\operatorname{Col}(A) = \left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \\ -3 \end{bmatrix} \right\}$.

$$\operatorname{Nul} A = \left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -4 \\ -3 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ -4 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

(2) ① If $w_1, w_2 \in T(U)$, then $w_1 = T(u_1)$, $w_2 = T(u_2)$ for some $u_1, u_2 \in U$.
Then $w_1 + w_2 = T(u_1) + T(u_2) = T(u_1 + u_2) \in T(U)$ since $u_1 + u_2 \in U$ as U is a subspace.

② If $w \in T(U)$, $c \in \mathbb{R}$, then $w = T(u)$ for some $u \in U$.
And $cw = cT(u) = T(cu) \in T(U)$ since $c \cdot u \in U$ as U is a subsp.

① & ② $\Rightarrow$ $T(U) \subset W$ subspace

(Note: checking $\vec{0}_W \in T(U)$ follows from ① or ②).