

MATH 54 SPRING 2019: DISCUSSION 109/112 QUIZ#2

GS1: CHRISTOPHER EUR, DATE: 2/12/2019

STUDENT NAME: It's too warm!

Problem 1. (5 points) Determine whether the following matrix A is invertible

$$A = \begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 1 \\ 4 & 1 & 1 \end{bmatrix}$$

Problem 2. (5 points) Let A and B be matrices such that AB is well-defined. Show that if the columns of B are linearly dependent, then so are the columns of AB .

(1) Sol 1: row reduce $\begin{bmatrix} 2 & 1 & -1 \\ 3 & 0 & 1 \\ 4 & 1 & 1 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & 1 & -1 \\ 0 & -3 & 5 \\ 0 & -1 & 3 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 2 & 1 & -1 \\ 0 & -1 & 3 \\ 0 & 0 & -4 \end{bmatrix}$

pivot in each row & col \Leftrightarrow invtbl.

Sol 2: $\det(A) = 2 \begin{vmatrix} 0 & 1 \\ 1 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 1 \\ 4 & 1 \end{vmatrix} - 1 \begin{vmatrix} 3 & 0 \\ 4 & 1 \end{vmatrix}$
 $= -2 + 1 - 3 = -4 \neq 0 \Rightarrow$ invtbl.

(2) B has lin. dep. col. $\Leftrightarrow B\vec{z} = \vec{0}$ for some nonzero \vec{z} .

\downarrow

$AB\vec{z} = A\vec{0} = \vec{0}$. Hence AB also lin. dep. col.