Quiz #7; Wed, 3/9/2016

Math 53 with Prof. Stankova

Section 110; MWF12-1

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Problem. Find three positive numbers whose *product* is 27 and whose *sum* is a *minimum.* (12 points for finding the numbers that *locally* minimize the sum; 3 points for justifying that it is indeed a global minimum).

Solution. We are looking at $x, y, z > 0$ such that $xyz = 27$, and minimizing $x+y+z$. Let $f(x,y) = x+y+27/xy$. We are minimizing f in the domain $x, y > 0$. We compute that $\nabla f = \langle 1-\frac{27}{x^2y}, 1-\frac{27}{xy^2}\rangle$ so that the critical point is $x = y = 3$. Second derivative test confirms that it is indeed a local maximum. To see that this is indeed a global minimum, note that if $x > 3$ and $y > 3$ then $f_x > 0$ and $f_y > 0$, so f increases as x, y increase outside the region $D = \{0 < x \leq 3,\ 0 < y \leq 3\}$. In D, along the boundary $x = 3$ or $y = 3$, the minimum is achieved at $(3, 3)$ again, so that the absolute minimum is indeed $(x, y, z) = (3, 3, 3)$.