Quiz #7; Wed, 3/9/2016

Math 53 with Prof. Stankova

Section 110; MWF12-1

GSI: Christopher Eur

Student Name: ______________________________

Problem. Let $f(x, y) = x^2 + y^2$, and let $\mathbf{r}(t) = \langle x(t), y(t), f(x(t), y(t))\rangle$ be a curve on the surface $z = f(x, y)$ where $x(t) = t\cos t$ and $y(t) = t\sin t$. Show that the tangent *line* L to the curve $r(t)$ at the point $(-\pi, 0, \pi^2)$ (i.e. $t = \pi$) is contained in the tangent *plane* T to the surface $z = f(x, y)$ at $(-\pi, 0, \pi^2)$.

Solution. $r'(t) = (\cos t - t\sin t, \sin t + t\cos t, 2t)$, and so the vector for the line is $r'(\pi) = (-1, -\pi, 2\pi)$. The tangent plane to the surface equation is $z - \pi^2 = -2\pi(x + \pi)$. Now, note that $\langle -1, \pi, 2\pi \rangle \cdot \langle -2\pi, 0, -1 \rangle = 0$. Since the tangent plane and the line both go through the point $(-\pi, 0, \pi^2)$, and the line is orthogonal to do normal vector of the plane, it is contained in the plane.