Quiz #2; Wed, 2/3/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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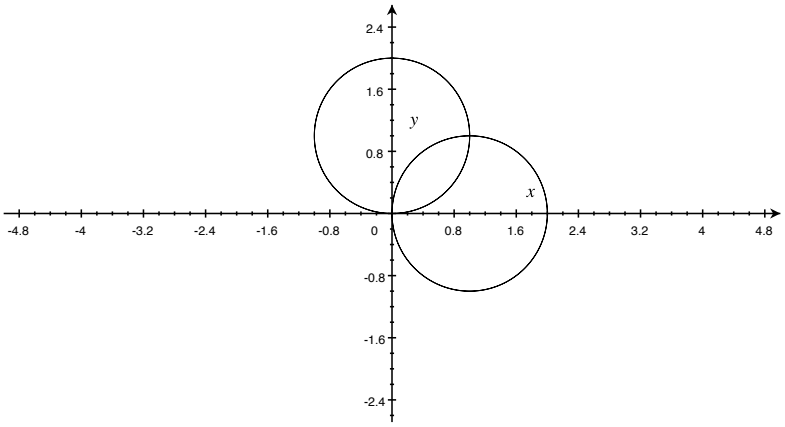
Problem. Sketch the two curves, and find the area of the region that lies inside at least one of the two curves (that is, the area of the union of the two enclosed regions):

$$r = 2\cos\theta, \qquad r = 2\sin\theta$$

Solution. These are the familiar circles. So the sketch of the two curves are:



Let's call the region enclosed by the first curve R_1 and the region enclosed by the second curve R_2. The lower half of $R_1 \cap R_2$ has area

$$\int_0^{\pi/4} \frac{1}{2}(4\sin^2\theta)d\theta = \int_0^{\pi/4} 2\sin^2\theta d\theta = \int_0^{\pi/4} 1 - \cos 2\theta d\theta = \frac{\pi}{4} - \frac{1}{2}$$

So, the area of $R_1 \cap R_2$ is $\frac{\pi}{2} - 1$, and hence the total area is

$$\text{Area}(R_1 \cup R_2) = \text{Area}(R_1) + \text{Area}(R_2) - \text{Area}(R_1 \cap R_2) = \pi + \pi - \frac{\pi}{2} + 1 = \frac{3\pi}{2} + 1$$