Quiz #1; Wed, 1/27/2016

Math 53 with Prof. Stankova

Section 110; MWF11-12

GSI: Christopher Eur

Student Name: ______________________________

Problem. (a) (12 points) Draw the curve on the x, y–plane defined by the following equation in polar coordinates:

$$\cos^2\theta = \frac{4}{r^2} - 4\sin^2\theta$$

(b) (3 points) Then find $\frac{dx}{dy}$ (Caution: NOT $\frac{dy}{dx}$) at $r = 2, \theta = 0$. (Hint: you may not need to do any computation if you have done part (a)).

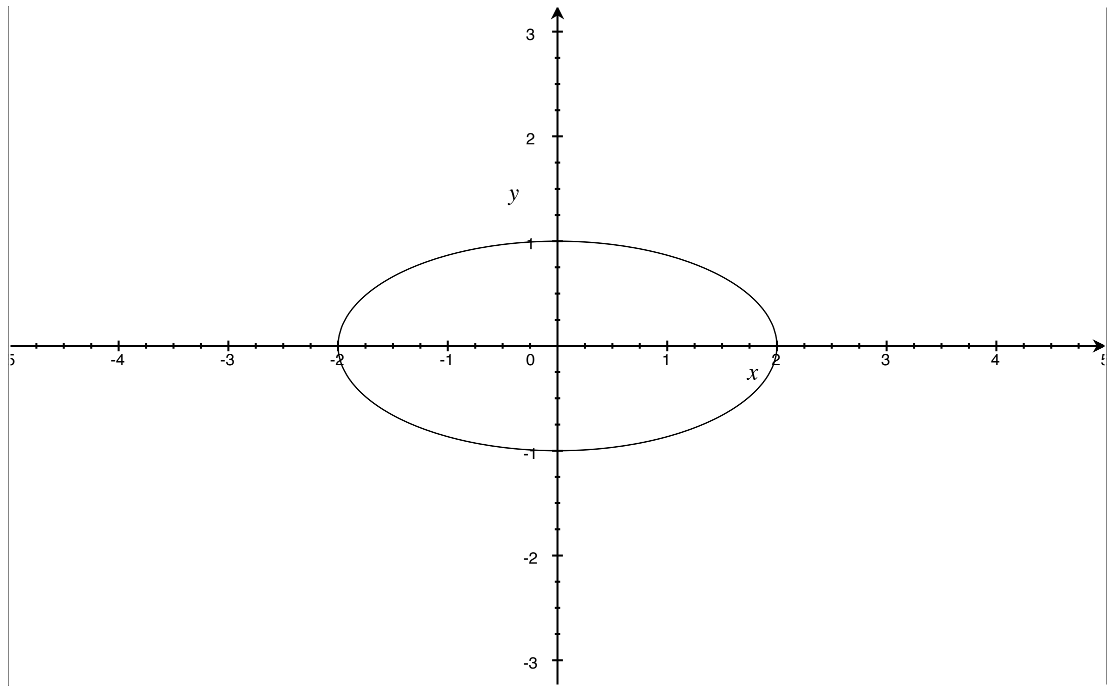
Solution. Multiplying $\frac{r^2}{4}$ on both sides and then simplifying, we get

$$\left(\frac{r\cos\theta}{2}\right)^2 = 1 - (r\sin\theta)^2$$

Thus, via the coordinate change $x = r\cos\theta,\ y = r\sin\theta$, we get

$$\frac{x^2}{2^2} + y^2 = 1$$

So, it is an ellipse:



(b) dy/dx is the slope of the tangent line to a given point. Here, we see that the tangent line to the curve at $r = 2, \theta = 0$ is the vertical line, and so dy/dx is infinite, in other words, $dx/dy = 0$.

Alternatively, one can compute dx/dy by

$$\begin{aligned}\frac{dx}{dy} &= \frac{dx/d\theta}{dy/d\theta} \\ &= \frac{\frac{dr}{d\theta}\cos\theta - r\sin\theta}{\frac{dr}{d\theta}\sin\theta + r\cos\theta} \\ &= \frac{1}{2}\left.\frac{dr}{d\theta}\right|_{r=2,\theta=0} \qquad (\because \text{ plug in } r = 2, \theta = 0)\end{aligned}$$

Implicitly differentiating $\cos^2\theta = \frac{4}{r^2} - 4\sin^2\theta$ by $d/d\theta$, and plugging in $r = 2, \theta = 0$, one obtains 0, as expected.