Differential Inclusions and the Control of Forest Fires

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Abstract: The talk will describe a new class of variational problems, motivated by the control of forest fires. The area burned by the fire (or contaminated by a spreading agent) at time $t > 0$ is modelled as the reachable set for a differential inclusion $\dot{x} \in F(x)$, starting from an initial set R_0. We assume that the spreading of the contamination can be controlled by constructing walls. In the case of a forest fire, one may think of a thin strip of land which is either soaked with water poured from above (by airplane or helicopter), or cleared from all vegetation using a bulldozer.

The first part of the talk will examine under which conditions there exists a strategy that blocks the fire within a bounded domain.

Next, consider a function $\alpha(x)$ describing the unit value of the land at the location x, and a function $\beta(x)$ accounting for the cost of building a unit length of wall near x. This leads to an optimization problem, where one seeks to minimize the total value of the burned region, plus the cost of building the barrier.

A general theorem on the existence of optimal strategies will be presented, together with various necessary conditions for optimality.

References:

1. Alberto Bressan, Differential inclusions and the control of forest fires, *J. Differential Equations* (special volume in honor of A. Cellina and J. Yorke), **243** (2007), 179-207.

2. Alberto Bressan and Camillo De Lellis, Existence of optimal strategies for a fire confinement problem, in preparation.