Random Discrete Structures

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http://www.math.cmu.edu/ af1p/Teaching/ATIRS/ATIRS.html

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$H_{n,m,k}$ is a random k-uniform hypergraph.

Vertex set is $\{1, 2, \dots, n\}$ and edge set is $\{E_1, E_2, \dots, E_m\}$, with $\forall i.\ |E_i| = k$.

** Something about perfect matchings *****

What we really want is m_0 such that

$$m \geq (1+\varepsilon)m_0 \implies \Pr(\cdots) \to 1$$
$$\text{and} \qquad m \leq (1-\varepsilon)m_0 \implies \Pr(\cdots) \to 0$$

Solved in 1960s.

$$m = \frac{n}{2}\left[\log n + c\right] \implies \Pr(\cdots) \approx \mathrm{e}^{-\mathrm{e}^{-c}} \qquad \text{(Erdös \& Renyi)}$$

Shamir & Schmidt-Pruzan:

$$k = 3 \qquad m \gg n^{3/2} \implies \exists \text{ p.m. w.h.p}$$

$H_{n,r,k}$ is a random k-uniform, r-regular hypergraph.

JKV: $m \geq Kn \log n \implies \exists$ p.m. w.h.p.

$K = ???$

Conjecture: $K = \frac{1}{k}$???

$H_0 := K_{n,k}$ is complete k-uniform hypergraph.

Define $H_0, H_1, H_2, \cdots, H_t$ by $H_{i+1} = H_i - \{\text{random edge}\}$, where $t = \binom{n}{k} - Kn \log n$.

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More details here

$H_{k,n}$ is complete $k-$uniform hypergraph.

$e_1, e_2, \ldots, e_{N=\binom{n}{k}}$ is a random ordering of the edges.

$E_i = H_{k,n} - \{e_1, e_2, \ldots, e_i\}$

$\Phi(H_i)$ is the number of perfect matchings of H_i

If $i \leq N - Kn\log n$, then $\Phi(H_i) \neq 0$ w.h.p. (Note: K is a large constant.)

$\mathcal{F}_i$ is set of factors.

$$|\mathcal{F}_i| = |\mathcal{F}_0|\frac{|\mathcal{F}_1|}{|\mathcal{F}_0|}\cdots\frac{|\mathcal{F}_i|}{|\mathcal{F}_{i+1}|} = |\mathcal{F}_0|(1-\xi_1)(1-\xi_2)\cdots(1-\xi_i)$$

$$\log|\mathcal{F}_t| = \log|F_0| + \sum_{i=1}^{t}\log(1-\xi_i)$$

$$\xi_i = \frac{|\Phi(H_i - \{e_i\})|}{\Phi(H_i)}$$

$$E(\xi_i) = \frac{n/k}{N-i+1} \leq \frac{1}{Kk\log n}$$

$$\sum \gamma_i = \frac{k-1}{k}n\log n - \frac{n}{k}\log\log n + O(n)$$

$w : A \to [0,\infty)$, $w_i(Z) = \Phi(H_i - Z)$.

$$\overline{w}(a) = \frac{1}{|A|}\sum_{a\in A} w(a)$$

$$\max w(A) = \max_{a\in A} w(a)$$

$$\text{maxr}\, w(A) = \frac{\max\ w(A)}{\overline{w}(A)}$$

$$\mathcal{C}_i = \left\{\max w_i(V_{k,Y}) \leq \max\{n^{-(k+1)}\Phi(H_i), 2\text{med}\, w_i(V_{k,Y})\}\ \forall Y \in V_{k-1}\right\}$$

$$\mathcal{A}_i\mathcal{R}_i\overline{\mathcal{B}_i} \subseteq \underbrace{(\mathcal{A}_i\mathcal{R}_i\overline{\mathcal{C}_i})}_{\text{small}} \cup \underbrace{(\mathcal{R}_i\mathcal{C}_i\overline{\mathcal{B}_i})}_{\text{small}}$$

$\mathcal{R}_i\mathcal{C}_i\overline{\mathcal{B}_i}$: $|Y| = k-1$;

$$\#y : w_i(Y+y) = \Omega(n^{-(k+1)}\Phi) \geq \frac{n-k}{2}$$