Department of Mathematics
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21-256 Multivariate Analysis
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Review Problems for Test 3

1. For each improper integral below, determine whether or not it converges. Evaluate (if possible) the convergent integrals.
   (a) $\int_0^\infty xe^{-5x}\,dx$
   (b) $\int_0^\infty \frac{dx}{1+e^x}$
   (c) $\int_0^1 \frac{dx}{\sqrt{4x^4-\sin^6 x}}$
   (d) $\int_0^1 \frac{x\,dx}{x^2-1}$
2. Determine whether or not the improper integral $\int_0^\infty \frac{dx}{\sqrt{1+e^x}}$ converges.
3. Evaluate the following double integrals:
   (a)
   $$\int_0^1 \int_0^{x^3} e^{y/x}\,dydx$$
   (b)
   $$\int_0^1 \int_{\sqrt{y}}^{2-\sqrt{y}} xy\,dxdy$$
   (c)
   $$\int_0^8 \int_{x^{1/3}}^2 \frac{dydx}{y^4+1}$$
4. Evaluate the given integrals by converting to polar coordinates:
   (a)
   $$\int_0^2 \int_0^{\sqrt{4-x^2}} x(x^2+y^2)^{3/2}dydx$$
   (b)
   $$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \ln(x^2+y^2+1)dxdy$$
5. Find the volume under the paraboloid $z = 3x^2 + y^2$ and above the region bounded by $y = x$ and $x = y^2 - y$.

6. Let D be the region in the plane bounded by $x = y^2$ and $x = 3 - 2y^2$. Let $f(x, y)$ be some continuous function. Fill in the four blank limits of integration to make the following equation correct:

$$\int_D \int f(x,y)dA = \int_{\square}^{\square} \int_{\square}^{\square} f(x,y)dx\,dy.$$

7. Evaluate the following triple integrals:

   (a)
$$\int_0^1 \int_0^{x^2} \int_0^{x+y} (2x - y - z)\,dzdydx$$

   (b)
$$\int_1^e \int_1^x \int_0^z \frac{2y}{z^3}\,dydzdx$$

8. Find the volume of the solid bounded by the paraboloids $z = 3x^2 + 3y^2$ and $z = 4 - x^2 - y^2$.

9. Use cylindrical coordinates to evaluate the integral

$$\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{-(x^2+y^2)}^{(x^2+y^2)} 21xy^2\,dzdydx$$

10. Fill in the seven boxes below to make the equation correct.

    Do NOT compute the integral

    Let E be the three dimensional volume bounded below by the surface $z = \sqrt{x^2 + y^2}$ and above by $x^2 + y^2 + z^2 = 9$.

$$\int\int_E\int xz\,dV = \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \boxed{}\,d\rho\,d\theta\,d\phi$$

    where $x = \rho\cos(\theta)\sin(\phi)$, $y = \rho\sin(\theta)\sin(\phi)$ and $z = \rho\cos(\phi)$.

11. Fill in the seven boxes below to make the equation correct.

    Do NOT compute the integral.

$$\int_0^{\pi/4} \int_0^{\sec(\theta)} \int_{r^2\cos^2(\theta)}^{r^2} r^2 z\cos(\theta)\,dz\,dr\,d\theta$$

$$= \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \boxed{}\,dz\,dy\,dx$$

    where $x = r\cos(\theta)$, $y = r\sin(\theta)$ (and $z = z$).

12. Fill in the six boxes below to make the equation correct.

    Do NOT compute the integral.

Let E be the solid tetrahedron with vertices $(0,0,0)$, $(0,2,0)$, $(1,1,0)$, $(2,0,2)$. This tetrahedron is bounded by the planes $z=0$, $z=x$, $z=x-y$, and $x+y=2$.

$$\iiint_E f(x,y,z)\,dV = \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} f(x,y,z)\,dy\,dx\,dz$$

13. From the choices below circle the one which equals

$$\int\int_E\int f(x,y,z)dV,$$

where E is the volume bounded by the planes $y=0$, $y=3$, $z=x$, $x=0$, and $x+z=2$.

(a)

$$\int_0^2\int_0^3\int_0^{2-z} f(x,y,z)dxdydz$$

(b)

$$\int_0^2\int_0^z\int_0^3 f(x,y,z)dydxdz$$

(c)

$$\int_0^3\int_0^1\int_x^{2-x} f(x,y,z)dzdxdy$$

(d)

$$\int_0^1\int_{2-x}^x\int_0^3 f(x,y,z)dydzdx$$

14. Evaluate

$$\int\int_E\int z(x^2+y^2+z^2)^{3/2}dV$$

where E is the volume bounded below by the cone

$$z=\sqrt{x^2+y^2}$$

and above by the sphere

$$x^2+y^2+z^2=4.$$

15. Let E be the solid bounded by the planes $y=3x$, $x+z=2$, $y=0$, and $z=0$. Fill in the six blank limits of integration so that

$$\int\int_E\int f(x,y,z)dV = \int_{\square}^{\square}\int_{\square}^{\square}\int_{\square}^{\square} \boxed{}\; f(x,y,z)dxdzdy.$$

16. The standard deviation for a random variable X with probability density function f and mean μ is defined by

$$\sigma = \sqrt{\int_{-\infty}^{\infty}(x-\mu)^2 f(x)\,dx}.$$

Suppose that f is an exponential density function, i.e.

$$f(x) = \begin{cases} \frac{1}{\mu}e^{-x/\mu} & x \geq 0 \\ 0 & x < 0 \end{cases}.$$

(a) Find the standard deviation for f.

(b) Find the probability that the random variable lies within one standard deviation of μ.

17. According to Big-Ears Real Estate Company, the prices of houses on the market in Toytown are exponentially distributed with mean £20,000, and the number of houses available for sale is exponentially distributed with mean 20. (Assume that the prices of houses and the number of houses for sale are independent random variables.)

    Noddy wants to buy a house that costs between £10,000 and £25,000. What is the probability of Noddy finding at least one house within his price range?