Department of Mathematics
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21-256 Multivariate Analysis
October 6, 2005

Review Problems for Test 1

1. Given the vectors $\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$, $\mathbf{b} = \mathbf{i} - 2\mathbf{k}$, and $\mathbf{c} = -\mathbf{i} + 4\mathbf{j} + 3\mathbf{k}$, compute the following expressions.
   (a) $\mathbf{a} \times \mathbf{b} + \mathbf{c}$
   (b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c} - 2 \mid \mathbf{c} \mid^2 \mathbf{a}$
   (c) $(\mathbf{a} \times \mathbf{b}) \times (\mathbf{c} \times \mathbf{a})$

2. Consider the matrices

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & 0 & -2 & 3 \\ 2 & -1 & 0 & 4 \\ 2 & 1 & -1 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} -1 & 0 & 5 \\ 0 & -3 & 0 \\ -1 & -4 & 2 \\ 1 & -1 & 2 \end{bmatrix}$$

   (a) Find $\det(A)$
   (b) Find $2AB - A^T B$

3. Using the inverse of the matrix of coefficients, determine the solution of the system of equations

$$\begin{cases} 3x + 2y = 2 \\ 2x - 2y = 3 \end{cases}$$

4. Consider the vectors $\mathbf{a}^T = [-1, 2, 3]$, $\mathbf{b}^T = [0, 1, -3]$, $\mathbf{c}^T = [2, -4, 2]$, $\mathbf{d}^T = [2, 0, -5]$.
   (a) Determine whether $\mathbf{a}, \mathbf{b}, \mathbf{c}$ are linearly independent.
   (b) Determine whether $\mathbf{b}, \mathbf{c}, \mathbf{d}$ are linearly independent.
   (c) Determine whether $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d}$ are linearly independent.

5. Using Gaussian elimination, solve the following systems of equations.

(a) $$\begin{cases} x - 2y + 3z = 2 \\ -3x + y - z = 1 \\ -x - 3y + 5z = 3 \end{cases}$$

(b) $$\begin{cases} 3x + y - z + 3t = 2 \\ -x + 4y + 2z - 2t = -1 \\ 2x + 5y + z + t = 1 \end{cases}$$

(c) $$\begin{cases} 2x - y + z = 1 \\ -3x + y + 2z = -2 \\ -x + z = 0 \end{cases}$$

6. Find the lengths of the sides of the triangle ABC, where $A(-1, 2, 5)$, $B(4, 0, 3)$, and $C(3, -2, -1)$.
7. Find the angle between $\mathbf{a} = \mathbf{i} + 3\mathbf{k}$ and $\mathbf{b} = -2\mathbf{i} + 3\mathbf{j} + \mathbf{k}$.
8. Find the value of x that ensures that the vectors $\mathbf{a} = -\mathbf{i} + 3\mathbf{j} + x\mathbf{k}$ and $\mathbf{b} = 2x\mathbf{i} + \mathbf{j} + \mathbf{k}$ are orthogonal.
9. Find the area of the triangle ABC, where $A(-1, -2, 2)$, $B(4, -3, 1)$, and $C(3, -4, 2)$.
10. Find a vector orthogonal to the plane determined by the points $A(0, 2, -1)$, $B(-2, 3, 1)$, and $C(4, -1, 5)$.
11. Find the volume of the parallelepiped with adjacent edges AB, AC, and AD, where $A(0, 2, 1)$, $B(2, -3, 4)$, $C(-4, 5, 1)$, and $D(3, 2, 0)$.