Department of Mathematical Sciences
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21.260 Differential Equations
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SEPARATION OF VARIABLES

Consider a 1st order ODE of the form

$$\frac{dy}{dx} = g(x)h(y), \tag{1}$$

where g is continuous, and h is continuously differentiable.

- To find *all* the solutions of (1):
  1. Find all the real numbers y^* satisfying $h(y^*) = 0$. For each such y^*, the constant function $y = y^*$ is a solution of (1).
  2. Separate variables and integrate, i.e.

     $$\int \frac{dy}{h(y)} = \int g(x) \; dx + C, \tag{2}$$

     where C is a constant of integration. If possible, use (2) to express y explicitly in terms of x.

- To solve the initial-value problem

  $$\frac{dy}{dx} = g(x)h(y), \quad y(x_0) = y_0 :$$

  1. If $h(y_0) = 0$, then the constant function $y = y_0$ is the solution.
  2. If $h(y_0) \neq 0$, use the initial condition $y(x_0) = y_0$ to solve for the constant C in (2). If possible, express y explicitly in terms of x.

Note: It follows from the uniqueness theorem that for a solution of (1), either y is a constant (i.e. $h(y) \equiv 0$) or $h(y)$ never vanishes. This is why the procedure described above works.