*we will go over the solution on Monday 8-11pm at West Wing Cluster.

Concepts Review

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Complete the Square

Prove the following for all real numbers x, y such that $x \geq y$,

$$2x^2 + y^2 \geq 2xy + x + y - 1$$

Induction

Remember Fibonacci Numbers?

$$F_n = \begin{cases} 0 & \text{if } n = 0; \\ 1 & \text{if } n = 1; \\ F_{n-1} + F_{n-2} & \text{if } n > 1. \end{cases}$$

Now, prove for all natural number n,

$$\sum_{i=0}^{n} iF_i = nF_{n+2} - F_{n+3} + 2$$

Function

Prove that the following function $f : \mathbb{R} \to \mathbb{R}$ is bijective.

$$f(x) = (3(x+1)^3 + 5)^3$$

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Counting 2 ways

Prove the following by counting 2 ways.

$$\sum_{i=0}^{n} \binom{n}{i} 100^i = 101^n$$

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Probability

What's the probability of a 5 card poker hand that has the following properties?

- Has exactly three cards of the same rank, but does not have a pair (aka not full house)
- Includes all four suits.

Expectation

There are 8 questions on a concepts final. Suppose that you had no idea how to solve any of them. So your strategy for the exam was as follows:

1. Choose a problem at random.
2. Stare at it for 1 minute. Then repeat step 1.

What's the expected number of minutes until you will stare at **all** the questions?