21-128 and 15-151: Class 2

August 28, 2024

News and Notes

News and Notes

Please read sections 1.2 and 1.3 of Clive.

News and Notes

Please read sections 1.2 and 1.3 of Clive.

Please read “ProofWritingExpectations”, if you have not done so already.

News and Notes

Please read sections 1.2 and 1.3 of Clive.

Please read “ProofWritingExpectations”, if you have not done so already.

HW1 is due $9/4$ by 11pm through Gradescope. Problem 6 has been reworded to ensure the uniqueness of a.

News and Notes

Please read sections 1.2 and 1.3 of Clive.

Please read “ProofWritingExpectations”, if you have not done so already.

HW1 is due 9/4 by 11pm through Gradescope. Problem 6 has been reworded to ensure the uniqueness of a.

LaTeX/Gradescope session: 1-2pm this Saturday, 8/31, in Doherty 2315.

News and Notes

Please read sections 1.2 and 1.3 of Clive.

Please read "ProofWritingExpectations", if you have not done so already.

HW1 is due 9/4 by 11pm through Gradescope. Problem 6 has been reworded to ensure the uniqueness of a.

LaTeX/Gradescope session: 1-2pm this Saturday, 8/31, in Doherty 2315.

Proof-writing tips session: 2:15-4pm this Saturday, 8/31, in Doherty 2315.

Recalling The Previous Class

Recalling The Previous Class

Discuss with your peers what we did last time.

Recalling The Previous Class

Discuss with your peers what we did last time.

Tell me what we did last time!

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

In general, assumptions should be stated and variables should be quantified. We need to see logical deductions and the explanations to justify them.

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

In general, assumptions should be stated and variables should be quantified. We need to see logical deductions and the explanations to justify them.

We discussed Roots, Exponents, and Absolute Value

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

In general, assumptions should be stated and variables should be quantified. We need to see logical deductions and the explanations to justify them.

We discussed Roots, Exponents, and Absolute Value - emphasizing quantification.

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

In general, assumptions should be stated and variables should be quantified. We need to see logical deductions and the explanations to justify them.

We discussed Roots, Exponents, and Absolute Value - emphasizing quantification.

We played The Finger Game

Recalling The Previous Class

Discuss with your peers what we did last time.
Tell me what we did last time!

We discussed the syllabus.

We had a discussion of the contents of Chapter 0.

We found all $x \in \mathbb{R}$ such that $\sqrt{x-3} = x-5$.

In general, assumptions should be stated and variables should be quantified. We need to see logical deductions and the explanations to justify them.

We discussed Roots, Exponents, and Absolute Value - emphasizing quantification.

We played The Finger Game - probability and statistics are big drivers in Science today.

Elements of First Order Logic

Elements of First Order Logic

A Propositional Variable is

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are,

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols,

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, "not", "and", "or", "implies", "iff". In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$,

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$,

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, "not", "and", "or", "implies", "iff". In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$,

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, "not", "and", "or", "implies", "iff". In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$, and $p \Leftrightarrow q$.

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$, and $p \Leftrightarrow q$.

A Propositional Formula is

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, “not”, “and”, “or”, “implies”, “iff”. In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$, and $p \Leftrightarrow q$.

A Propositional Formula is an expression that is either a propositional variable, or is “built up” from simpler propositional formulae using a logical operator.

Elements of First Order Logic

A Propositional Variable is a variable that assumes the values True and False.

The Logical Connectives are, "not", "and", "or", "implies", "iff". In symbols, $\neg$, $\wedge$, $\vee$, $\Rightarrow$, $\Leftrightarrow$.

Recall the conditions for the truth of $\neg p$, $p \wedge q$, $p \vee q$, $p \Rightarrow q$, and $p \Leftrightarrow q$.

A Propositional Formula is an expression that is either a propositional variable, or is "built up" from simpler propositional formulae using a logical operator.

Observe that **parenthesization** of propositional formulae is, in general, necessary.

Elements of First Order Logic (continued)

Elements of First Order Logic (continued)

$\forall$ means

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means

Elements of First Order Logic (continued)

$\forall$ means “for every” and $\exists$ means “there exists”.

$\exists!$ means “ there exists a unique”.

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means " there exists a unique".

A Predicate is

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means " there exists a unique".

A Predicate is a symbol p with variables $x_1, \ldots, x_n$ coming from specified sets which evaluates to True or False when $x_1, \ldots, x_n$ are all known.

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists !$ means " there exists a unique".

A Predicate is a symbol p with variables $x_1, \ldots, x_n$ coming from specified sets which evaluates to True or False when $x_1, \ldots, x_n$ are all known.

A Logical Formula is

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means " there exists a unique".

A Predicate is a symbol p with variables $x_1, \ldots, x_n$ coming from specified sets which evaluates to True or False when $x_1, \ldots, x_n$ are all known.

A Logical Formula is a predicate with some (possibly all) of the variables bound by quantifiers. The sets for free variables should be displayed for the reader.

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means " there exists a unique".

A Predicate is a symbol p with variables $x_1, \ldots, x_n$ coming from specified sets which evaluates to True or False when $x_1, \ldots, x_n$ are all known.

A Logical Formula is a predicate with some (possibly all) of the variables bound by quantifiers. The sets for free variables should be displayed for the reader.

"s.t." is

Elements of First Order Logic (continued)

$\forall$ means "for every" and $\exists$ means "there exists".

$\exists!$ means " there exists a unique".

A Predicate is a symbol p with variables $x_1, \ldots, x_n$ coming from specified sets which evaluates to True or False when $x_1, \ldots, x_n$ are all known.

A Logical Formula is a predicate with some (possibly all) of the variables bound by quantifiers. The sets for free variables should be displayed for the reader.

"s.t." is shorthand for "such that".

Proving a Theorem

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

It helps to understand all of the **definitions** and **quantification** in the statement of the theorem.

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

It helps to understand all of the **definitions** and **quantification** in the statement of the theorem.

Ask yourself if you can write down some examples or evidence for the truth of the theorem in an effort to gain motivation.

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

It helps to understand all of the **definitions** and **quantification** in the statement of the theorem.

Ask yourself if you can write down some examples or evidence for the truth of the theorem in an effort to gain motivation. Do you believe the theorem to be a true statement?

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

It helps to understand all of the **definitions** and **quantification** in the statement of the theorem.

Ask yourself if you can write down some examples or evidence for the truth of the theorem in an effort to gain motivation. Do you believe the theorem to be a true statement?

If you change the quantifying sets does it become easier to prove?

Proving a Theorem

When presented with a theorem to prove, half of the difficulty can be understanding what the theorem says and what there is to prove, let alone formulating a strategy for a proof.

It helps to understand all of the **definitions** and **quantification** in the statement of the theorem.

Ask yourself if you can write down some examples or evidence for the truth of the theorem in an effort to gain motivation. Do you believe the theorem to be a true statement?

If you change the quantifying sets does it become easier to prove? Does it become false?

Proving a Theorem (continued)

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe?

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe? We observe that the sequences are interlaced,

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe? We observe that the sequences are interlaced,

1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49,$\ldots$

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe? We observe that the sequences are interlaced,

1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49,...

the products fall strictly between x^2 and $(x+1)^2$,

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe? We observe that the sequences are interlaced,

1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49,...

the products fall strictly between x^2 and $(x+1)^2$, so they are never squares.

Proving a Theorem (continued)

Theorem: The product of any two consecutive positive integers is never a perfect square.

Motivation for a proof: Write out the sequence of positive squares, and the sequence of products of consecutive positive integers.

$1, 4, 9, 16, 25, 36, 49, \ldots$ and $2, 6, 12, 20, 30, 42, \ldots$

What do you observe? We observe that the sequences are interlaced,

1, 2, 4, 6, 9, 12, 16, 20, 25, 30, 36, 42, 49,...

the products fall strictly between x^2 and $(x+1)^2$, so they are never squares. We need to formalize this.

Proving a Theorem (continued)

Proving a Theorem (continued)

Proof:

Proving a Theorem (continued)

Proof: Let x and y be positive integers.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Case 2) If $y > x$, then $y \geq x+1$,
so $y^2 \geq (x+1)^2 > x(x+1)$.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Case 2) If $y > x$, then $y \geq x+1$,
so $y^2 \geq (x+1)^2 > x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Case 2) If $y > x$, then $y \geq x+1$,
so $y^2 \geq (x+1)^2 > x(x+1)$.

Thus, $x(x+1)$ is not y^2. Since the two cases exhaust all possibilities, our proof is complete.

Proving a Theorem (continued)

Proof: Let x and y be positive integers.
We need to show that $x(x+1)$ is not y^2.

Case 1) If $y \leq x$, then $y^2 \leq x^2 < x(x+1)$.

Thus, $x(x+1)$ is not y^2.

Case 2) If $y > x$, then $y \geq x+1$,
so $y^2 \geq (x+1)^2 > x(x+1)$.

Thus, $x(x+1)$ is not y^2. Since the two cases exhaust all possibilities, our proof is complete.

Theorem (proved in 1974 by Erdos and Selfridge):
The product of two or more consecutive positive integers is never a square or any other higher power.

Proving another Theorem

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof:

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof: Try to write some small, odd primes as the difference of squares.

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof: Try to write some small, odd primes as the difference of squares. We see that $3 = 2^2 - 1^2$, $5 = 3^2 - 2^2$ and $7 = 4^2 - 3^2$.

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof: Try to write some small, odd primes as the difference of squares. We see that $3 = 2^2 - 1^2$, $5 = 3^2 - 2^2$ and $7 = 4^2 - 3^2$.

Proof:

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof: Try to write some small, odd primes as the difference of squares. We see that $3 = 2^2 - 1^2$, $5 = 3^2 - 2^2$ and $7 = 4^2 - 3^2$.

Proof: Let $p \geq 3$ be a prime number.

Proving another Theorem

Theorem: For every prime $p \geq 3$, there exist unique $n, m \in \mathbb{N}$ such that

$$p = n^2 - m^2.$$

Motivation for a proof: Try to write some small, odd primes as the difference of squares. We see that $3 = 2^2 - 1^2$, $5 = 3^2 - 2^2$ and $7 = 4^2 - 3^2$.

Proof: Let $p \geq 3$ be a prime number.

Existence: One can verify that $n = (p+1)/2$ and $m = (p-1)/2$ are natural numbers such that $p = n^2 - m^2$.

Proving another Theorem (continued)

Proving another Theorem (continued)

Uniqueness:

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

We have $p = (n-m)(n+m)$.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

We have $p = (n-m)(n+m)$. Since p is positive, $n-m$ and $n+m$ must be positive integers.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

We have $p = (n-m)(n+m)$. Since p is positive, $n-m$ and $n+m$ must be positive integers.

Since p is prime, it has only 1 and p as positive factors.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

We have $p = (n-m)(n+m)$. Since p is positive, $n-m$ and $n+m$ must be positive integers.

Since p is prime, it has only 1 and p as positive factors. Thus, $n-m = 1$, and $n+m = p$.

Proving another Theorem (continued)

Uniqueness: Assume that n and m are natural numbers such that $p = n^2 - m^2$.
We must deduce that $n = (p+1)/2$ and $m = (p-1)/2$.

We have $p = (n-m)(n+m)$. Since p is positive, $n-m$ and $n+m$ must be positive integers.

Since p is prime, it has only 1 and p as positive factors. Thus, $n-m = 1$, and $n+m = p$.

Solving two equations in two unknowns yields $n = (p+1)/2$ and $m = (p-1)/2$, as desired.

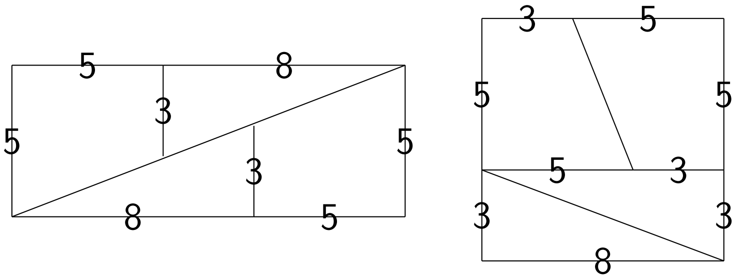
Is it possible that $64 = 65$?

Is it possible that $64 = 65$?

Consider the rearrangement of the 4 pieces from the left into the 4 pieces on the right:

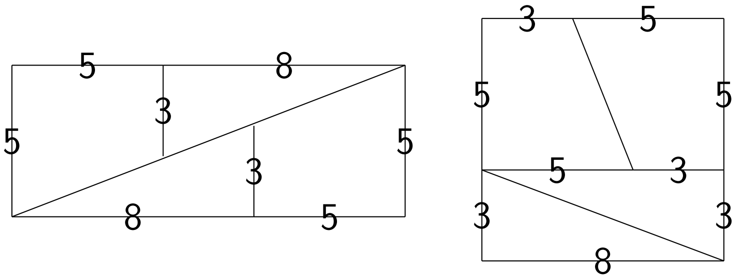
Is it possible that $64 = 65$?

Consider the rearrangement of the 4 pieces from the left into the 4 pieces on the right:



Is it possible that $64 = 65$?

Consider the rearrangement of the 4 pieces from the left into the 4 pieces on the right:



What is wrong with the argument given above?

It is not possible that $64 = 65$!

It is not possible that $64 = 65$!

The lines labeled with "3" should be labeled with "$\frac{40}{13}$". When reassembled on the right, the top rectangle is 5 by $\frac{105}{13}$ and the bottom rectangle is 8 by $\frac{40}{13}$. The top and bottom rectangles don't form a square.

Calculate the Area of a Square

Calculate the Area of a Square

What is the area of the inner square in the following diagram?

Calculate the Area of a Square

What is the area of the inner square in the following diagram?