FIELDS HOMEWORK SET V

JAMES CUMMINGS

You may collaborate on this homework set, but must write up your solutions by yourself. Please contact me by email if you are puzzled by something, would like a hint or believe that you have found a typo.

Due March 31st.

(1) Let F be an extension of E, let $\rho \in Aut(F/E)$ and $H \leq Aut(F/E)$. What is the relationship between $Fix(H)$ and $Fix(H^\rho)$?

(2) Complete the calculation (mostly done in class) of all the intermediate fields for the splitting field of $x^5 - 2$ over $\mathbb{Q}$.

(3) Let E be the subfield of the complex numbers generated over $\mathbb{Q}$ by the roots of $x^4 + 2$. Compute the degree $[E : \mathbb{Q}]$. Find all the intermediate fields of this extension. Which ones are Galois extensions of $\mathbb{Q}$?

(4) Let p be prime, let $E = \mathbb{Z}/p\mathbb{Z}$ and let $f = x^{p^n} - x$ for some $n > 0$. Prove that f has no repeated roots in any extension of E. Prove that if F is a splitting field for f over E then the roots of f form a subfield of F and use this to conclude that $|F| = p^n$.

(5) Let F be an algebraic extension of E. Prove that the set of $a \in F$ which are separable over E forms a subfield of F.