COMMUTATIVE ALGEBRA HW 5

JC

Due in class Wed 14 September.

(1) Let R be a UFD. Show that $R[x]$ has infinitely many irreducibles.
(2) Let K be a field, let $n \geq 1$ and let $K(x_1, \ldots x_n)$ be the field of fractions of the polynomial ring $K[x_1, \ldots x_n]$. Show that $K(x_1, \ldots x_n)$ is not ring-finite/K. Hint: $K[x_1, \ldots x_{n-1}]$ is a UFD.
(3) We proved in class that if $A \leq B \leq C$ are rings with A Noetherian and C both module-finite/B and ring-finite/A then B is ring-finite/A, Show that this is false if we drop the hypothesis that A is Noetherian.
(4) Let X be a topological space. We say that $A \subseteq X$ is *closed* iff its complement $X \setminus A$ is open. Given a ring R, identify the prime ideals P such that $\{P\}$ is closed in $Spec(R)$.