T-79.7003: Graphs and Networks **Out: Nov. 29th, 2013**

Homework 3

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3.1 1D RW [10 Points]

In class we saw the following system of equations when we discussed Papadimitriou's randomized algorithm for 2SAT: $x_0 = x_n = 1$ and for any $k \in \{1, \ldots, n-1\}$ $x_k = \frac{1}{2}(x_{k-1}+1) + \frac{1}{2}(x_{k+1}+1)$. Show that the system of equations has a *unique* solution.

3.2 Spectral Theorem [30 Points]

Suppose that A is a symmetric $n \times n$ real matrix. Show that A has the following properties: (A) The eigenvalues are all real. (B) A has a complete set of eigenvalues and eigenvectors, i.e., its eigenvectors span a space of dimension n. (C) If $\lambda_1, \ldots, \lambda_n$ are the eigenvalues of A and $x_1, \ldots, x_n$ the corresponding orthonormal eigenvectors as column vectors then

$$A = \sum_{i=1}^{n} \lambda_i x_i x_i^T.$$

3.3 Resistance metric [30 Points]

Let $G(V, E, c)$ be a graph. Show that effective resistance defines a metric on G.

3.4 A Classic Markov Chain [30 Points]

In an election where candidate A receives p votes and candidate B receives q votes with $p > q$, what is the probability that A will be strictly ahead of B throughout the count?