On the uniqueness of positive solutions of a quasilinear equation containing a weighted p-Laplacian, the superlinear case

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Abstract: We consider the quasilinear equation of the form

$$-\Delta_p u = K(|x|)f(u), x \in \mathbb{R}^n, n > p > 1,$$

where K is a positive C1 function defined in R^+, and $f \in C[0,\infty)$ has one zero at $u_0 > 0$, is non positive and not identically 0 in $(0, u_0)$, and is locally lipschitz, positive and satisfies some superlinear growth assumption in (u_0, ∞). We carefully study the behaviour of solutions of the corresponding initial value problem for the radial version of the quasilinear equation and combining, as Cortazar, Felmer, and Elgueta, comparison arguments due to Coffman and Kwong, which were thought to be restricted to the semilinear case only ($p = 2$), with some separation techniques, we show that any zero of the solutions to the initial value problem is monotone decreasing with respect to the initial value, which leads immediately the uniqueness of positive radial ground states, and the uniqueness of positive radial solutions of the Dirichlet problem on a ball.

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