Math 279R: Positivity in Matroid Theory

Instructor: Christopher Eur

Spring 2022

Time: Tues/Thur 10:30 – 11:45am   Location: SC 112

Office hours: TBD   Email: `ceur@math.harvard.edu`

Course website: `https://people.math.harvard.edu/~ceur/math279rs22.html`

Course description

Matroids are combinatorial objects that capture the essence of linear independence. Algebraic geometry has furnished fruitful tools for studying them, leading to resolutions of many long-standing conjectures. We survey these developments, starting with elementary matroid theory, and ending with the most recent breakthroughs. Along the way, we will meet various techniques in combinatorial algebraic geometry, such as toric geometry and tropical Hodge theory.

Grading

(For those who require a grade, e.g. undergraduate students and graduate students who have not passed the quals): Final grade will be based upon problem sets (40%), and a final project (60%). For the final project, the student can either (i) give a presentation on a relevant topic, or (ii) make a meaningful Macaulay2 code contribution for computations with matroids.

Prerequisites

Prerequisites can vary depending on one's goals. For a full appreciation of the course, some familiarity with algebraic geometry and algebraic topology at the level of a first graduate course is required. If unsure about the prerequisites for the course, please talk to the instructor.

Covid19 Contingency

Technology permitting, the lectures will be live-streamed over Zoom (but not recorded), at least for the first few weeks of the semester. The livestream is intended only for those who are unable to attend the lecture. When in such situation, please email the instructor for the Zoom link info.

List of topics

(Tentative, subject to change per audience interest and background)

Axioms and examples of matroids
Constructions: minors, duality, and quotients
Tutte polynomials and characteristic polynomials
Topology of hyperplane arrangement complements
Wonderful compactifications
Lorentzian polynomials
Toric and tropical intersection theory
Tropical Hodge theory
Matroids and Grassmannians
Tautological classes of matroids

References

Each lecture will be accompanied by lecture notes with detailed references that the lecture draws upon. Some standard references: matroid theory [Oxl11, Wel76], tropical geometry: [MS15], toric geometry [Ful93, CLS11]. Some helpful surveys, which inspired several aspects of this course, include [Bak18, Huh18, Kat16].

[Bak18] Matthew Baker. Hodge theory in combinatorics. *Bull. Amer. Math. Soc. (N.S.)*, 55(1):57–80, 2018.

[CLS11] David A. Cox, John B. Little, and Henry K. Schenck. *Toric varieties*, volume 124 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2011.

[Ful93] William Fulton. *Introduction to toric varieties*, volume 131 of *Annals of Mathematics Studies*. Princeton University Press, Princeton, NJ, 1993. The William H. Roever Lectures in Geometry.

[Huh18] June Huh. Tropical geometry of matroids. *Current Developments in Mathematics 2016*, pages 1–46, 2018.

[Kat16] Eric Katz. Matroid theory for algebraic geometers. pages 435–517, 2016.

[MS15] Diane Maclagan and Bernd Sturmfels. *Introduction to tropical geometry*, volume 161 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2015.

[Oxl11] James Oxley. *Matroid theory*, volume 21 of *Oxford Graduate Texts in Mathematics*. Oxford University Press, Oxford, 2 edition, 2011.

[Wel76] D. J. A. Welsh. *Matroid theory*. Academic Press [Harcourt Brace Jovanovich, Publishers], London-New York, 1976. L. M. S. Monographs, No. 8.