MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#9

GSI: CHRISTOPHER EUR, DATE: 10/27/2017

STUDENT NAME: K-theory

Problem 1. If true, prove the statement. If false, give a counterexample.

3pts (a) Suppose $u, v_1, v_2 \in \mathbb{R}^2$ such that $u \cdot v_1 = u \cdot v_2$. Then $v_1 = v_2$.

3pts (b) If a $n \times m$ matrix A has orthonormal columns, then $AA^T = I_n$.

4pts *Problem 2.* Show that if a $n \times m$ matrix consists of nonzero orthogonal columns, then $m \leq n$. (Hint: are the columns then linearly independent?)

#1. (a) Nope. Witness: $u = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $v_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $v_2 = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$. $(u \cdot v_1 = u \cdot v_2 = 0)$.

(b) Nope. Consider $A = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$. $AA^T = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \neq I_2$.

#2. Let A have columns $\vec{a}_1, \dots, \vec{a}_m \in \mathbb{R}^n$.

Claim: We'll show these form a lin. indep. set of vectors.
Then $\mathrm{nul}(A) = \{0\}$ implies that $m \leq n$ ($\because$ if $m > n$, then $\exists$ at least one free col. when A in RREF).

pf of Claim: Suppose $c_1 \vec{a}_1 + \dots + c_m \vec{a}_m = 0$ for some $c_1, \dots, c_m$ not all zero.

(You may also cite thm 1 of §6.1)

Let's say $c_i \neq 0$. Then $\vec{a}_i \cdot (c_1 \vec{a}_1 + \dots + c_m \vec{a}_m) = \vec{a}_i \cdot \vec{0} = 0$

$$\vec{a}_i \cdot (c_1 \vec{a}_1 + \dots + c_m \vec{a}_m) = c_i \|\vec{a}_i\|^2$$

($\vec{a}_i \cdot \vec{a}_j = 0$ for $i \neq j$ since A has orthogonal col.).

We have $c_i \|\vec{a}_i\|^2 = 0$ but $\vec{a}_i \neq 0 \Rightarrow \|a_i\|^2 \neq 0$,

Contradiction ※.

Tally:

A

B

20 > 19 A wins.