MATH 54 FALL 2017: DISCUSSION 205/208 QUIZ#4

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Problem 1. (6 points) Let A be a 2×3 matrix

$$A := \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & -3 \end{bmatrix}$$

(a): Find a basis for the nullspace $\text{nul}(A)$ of A.

(b): Find a basis for the column space $\text{col}(A)$ of A.

(c): Verify the Rank Theorem in this example. The Rank Theorem states: for an $m \times n$ matrix, one always has $\dim \text{col}(A) + \dim \text{nul}(A) = n$.

Problem 2. (4 points) Suppose $A\vec{x} = \vec{b}$ has a solution $\vec{x}_0$. Show that if $\vec{x}_1$ is another solution to $A\vec{x} = \vec{b}$, then $\vec{x}_1 = \vec{x}_0 + \vec{u}$ for some vector $u \in \text{nul}(A)$.

#1 $A \sim \begin{bmatrix} 1 & 2 & -2 \\ 0 & -1 & -1 \end{bmatrix} \Rightarrow \{ \vec{x} \mid A\vec{x} = \vec{0} \} = \left\{ \begin{bmatrix} 4t \\ -t \\ t \end{bmatrix} \mid t \in \mathbb{R} \right\}.$

$\sim \begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 1 \end{bmatrix}$

(a) $\text{nul}(A) = \text{span} \begin{bmatrix} +4 \\ -1 \\ +1 \end{bmatrix}$. basis: $\left\{ \begin{bmatrix} 4 \\ -1 \\ 1 \end{bmatrix} \right\}$.

(b) basis for col A: $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\}$.

(c)
$$\begin{array}{r} \dim \text{col}(A) = 2 \\ +)\ \dim \text{nul}(A) = 1 \\ \hline 3 = \dim \mathbb{R}^3 = 3 \end{array} \quad \checkmark$$

#2. $A(x_1 - x_0) = Ax_1 - Ax_0 = b - b = 0$. Hence $x_1 - x_0 \in \text{nul}(A)$.

Thus, setting $u = x_1 - x_0$, we get $x_1 = x_0 + u$ as desired.