MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#5

GSI: CHRISTOPHER EUR, DATE: 9/30/2016

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Problem 1. (a) (3 points) Compute the determinant of $A := \begin{bmatrix} 1 & 1 & 2 \\ -1 & 3 & -1 \\ 2 & -2 & 1 \end{bmatrix}$. Is A invertible?

(b) (3 points) *Using Cramer's rule*, solve the system of linear equations:

$$\begin{cases} 2x - y &= -1 \\ -x + y &= 1 \end{cases}$$

Problem 2. Let A be an invertible square matrix.

(a) (2 points) Show that $(\det A)(\det A^{-1}) = 1$.

(b) (2 points) Use part (a) to show the following statement:

*Suppose A is a matrix with all **integer entries** such that A^{-1} also has all integer entries. Then $|\det(A)| = |\det(A^{-1})| = 1$.*

[Hint: If A has all integer entries, is $\det A$ also an integer?]

(1) (a) $\begin{vmatrix} 1 & 1 & 2 \\ -1 & 3 & -1 \\ 2 & -2 & 1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ -1 & 4 & 1 \\ 2 & -4 & -3 \end{vmatrix} = 1 \begin{vmatrix} 4 & 1 \\ -4 & -3 \end{vmatrix} = -12 + 4 = -8 \neq 0.$

$\Downarrow$

A invtbl.

(b) $\begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$. $x = \dfrac{\begin{vmatrix} -1 & -1 \\ 1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}} = 0$, $y = \dfrac{\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 \\ -1 & 1 \end{vmatrix}} = \dfrac{1}{1} = 1.$

(2) (a) $(\det A)(\det A^{-1}) = \det(AA^{-1}) = \det I = 1.$

(b) A has integer entries. Laplace expansion makes it clear that $\det A$ is then also an integer. Same for A^{-1}.

Then $(\det A)(\det A^{-1}) = 1$ (from (a)) $\Rightarrow$ $\det A = \det A^{-1} = 1$

($\det A$: integer; $\det A^{-1}$: integer)

or $\det A = \det A^{-1} = -1$.

$\Rightarrow |\det A| = |\det A^{-1}| = 1$