MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#3

GSI: CHRISTOPHER EUR, DATE: 9/16/2016

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Problem 1. Let $f : \mathbb{R}^3 \to \mathbb{R}^2$ be a linear map defined by $f(x, y, z) = (-y + z, x - 3y + 2z)$, and let $g : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear map which is a $\pi/2$ radians rotation (counterclockwise) around the origin.

(a) (3 points) Write down the two matrices that correspond to f and g.

(b) (3 points) Find all (x, y, z) such that $g(f(x, y, z)) = (1, -1)$. If there is no such (x, y, z), then explain why.

Problem 2. Let $f : \mathbb{R}^m \to \mathbb{R}^n$ be a *linear* map throughout this question.

(a) (1 points) Give an example of f (i.e. write down the corresponding matrix) that is onto but not one-to-one (you should pick concrete values for m, n). You need not justify your answer.

(b) (1 points) Give an example of f that is one-to-one but not onto (again, you should pick concrete values for m, n). You need not justify your answer.

(c) (2 points) Now, suppose $m = n$, and f is one-to-one. Is it necessarily true that f is also onto? Why? [Hint: let A be the standard matrix of f. What can you say about A if f is one-to-one?]

1. (a) $[f] = \begin{bmatrix} 0 & -1 & 1 \\ 1 & -3 & 2 \end{bmatrix}$ 1.5, $[g] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ 1.5

(b) $[g][f]\vec{x} = \begin{bmatrix} -1 & 3 & -2 \\ 0 & -1 & 1 \end{bmatrix}\vec{x} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ 1 $\rightsquigarrow \left[\begin{array}{ccc|c} -1 & 3 & -2 & 1 \\ 0 & -1 & 1 & -1 \end{array}\right] \sim \left[\begin{array}{ccc|c} -1 & 0 & 1 & -2 \\ 0 & -1 & 1 & -1 \end{array}\right]$ 1

$\sim \left[\begin{array}{ccc|c} 1 & 0 & -1 & 2 \\ 0 & 1 & -1 & 1 \end{array}\right]$

$\boxed{\therefore\ z = \text{free},\ x = z+2,\ y = z+1}$ 1

$\hookrightarrow$ or, $\boxed{\vec{x} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}}$

2. (a) $\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$ 1 $m=3$ $n=2$ (b) $\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$ 1 $m=2$ $n=3$

(c) $[A]$ is $n \times n$. Since one-to-one, $[A]$ ~~[illegible]~~ when row reduced has 1 no free col, i.e. all col. become pivot col. Thus, row reduction of square matrix A produces $\begin{bmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{bmatrix}$, the identity matrix. 1 By Thm in 2.2, this means A invertible, and hence onto.