MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#13

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Problem 1. Find $u(x,t)$ satisfying the following (where $0 \le x \le \pi$ and $t \ge 0$):

$$u_t = u_{xx} \quad (*)$$

I.C. $u(x,0) = 4\cos\frac{5}{2}x - 3\cos\frac{9}{2}x$

B.C. $u(\pi,t) = 0$, $u_x(0,t) = 0$ (note the partial)

B.C. homogenous ✓

(1) Sep. of var: $u = X(x)T(t) \Rightarrow \frac{T'}{T} = \frac{X''}{X} = \lambda$ from $(*)$

(1) $X'' - \lambda X = 0$ Cases ① $\lambda > 0 \Rightarrow X = a e^{\sqrt{\lambda}x} + b e^{-\sqrt{\lambda}x}$

(1) Apply B.C. $\Rightarrow$ $a e^{\sqrt{\lambda}\pi} + b e^{-\sqrt{\lambda}\pi} = 0$

$a\sqrt{\lambda} - b(-\sqrt{\lambda}) = 0$, $\det\begin{bmatrix} e^{\sqrt{\lambda}\pi} & e^{-\sqrt{\lambda}\pi} \\ \sqrt{\lambda} & -\sqrt{\lambda} \end{bmatrix} < 0$

So $a = b = 0$ only solns.

② $\lambda = 0 \Rightarrow X = ax + b$. Again, B.C. $\Rightarrow a = b = 0$.

③ $\lambda < 0 \Rightarrow X = a\sin(\sqrt{-\lambda}x) + b\cos(\sqrt{-\lambda}x)$

(4) From $u_x(0,t) = 0$, get $a = 0$.

$u(\pi,t) = 0 \Rightarrow \cos(\sqrt{-\lambda}\pi) = 0$. want $\sqrt{-\lambda}\pi = n\pi + \frac{\pi}{2}$ $(n \in \mathbb{Z})$

$\therefore \sqrt{-\lambda} = n + \frac{1}{2}$

$\lambda = -(n+\frac{1}{2})^2$ for $n = 0, 1, 2, \ldots$

(2) (2) $T' - \lambda T = 0 \Rightarrow T = C_\lambda e^{\lambda t}$

Thus, $u(x,t) = \sum_{n=1}^{\infty} A_n \cos(n+\frac{1}{2})x \cdot e^{-(n+\frac{1}{2})^2 t}$ I.C. $\Rightarrow$ $A_2 = 4$, $A_4 = -3$, $A_i = 0$ $i \neq 2,4$

(2) $\therefore u(x,t) = 4(\cos\frac{5}{2}x)e^{-\frac{25}{4}t} - 3(\cos\frac{9}{2}x)e^{-\frac{81}{4}t}$