# MATH 54 FALL 2016: DISCUSSION 102/105 QUIZ#10

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STUDENT NAME: Pass

*Problem 1.* Consider a subspace $V \subset \mathbb{R}^4$ spanned by linearly independent set of vectors $v_1, v_2, v_3 \in \mathbb{R}^4$ given as follows:

$$v_1 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \; v_2 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix}, \; v_3 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 1 \end{bmatrix}$$

(a) (3 points) Carry out the Gram-Schmidt process on $(v_1, v_2, v_3)$ to obtain an orthogonal basis of $V$.

(b) (2 points) Compute $\mathrm{proj}_V \vec{y}$ where $\vec{y} = \begin{bmatrix} 1 \\ -1 \\ 3 \\ 2 \end{bmatrix}$

(a) $w_1 = v_1$, $\quad w_2 = v_2 - \frac{v_2 \cdot w_1}{w_1 \cdot w_1} w_1 = \begin{bmatrix} 2 \\ 0 \\ 0 \\ 2 \end{bmatrix} - \frac{4}{4} w_1 = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}$

$$w_3 = v_3 - \frac{v_3 \cdot w_1}{w_1 \cdot w_1} w_1 - \frac{v_3 \cdot w_2}{w_2 \cdot w_2} w_2 = \begin{bmatrix} 3 \\ -1 \\ 1 \\ 1 \end{bmatrix} - \frac{4}{4} w_1 - \frac{4}{4} w_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$$

$$\boxed{(w_1, w_2, w_3) = \left( \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} \right)}$$

(b) $\mathrm{Proj}_V \vec{y} = \frac{w_1 \cdot y}{w_1 \cdot w_1} w_1 + \frac{w_2 \cdot y}{w_2 \cdot w_2} w_2 + \frac{w_3 \cdot y}{w_3 \cdot w_3} \cancel{y} w_3$

$$= \frac{5}{4} \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix} + \frac{3}{4} \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix} = \boxed{\frac{1}{4} \begin{bmatrix} 9 \\ 1 \\ 7 \\ 3 \end{bmatrix}}$$

*Problem 2.* (1 point each) Fill in the blank in proving the following statement: *Let* $W \subset \mathbb{R}^n$ *be a subspace, then* $\dim W + \dim W^{\perp} = n$.

*Proof.* By Gram-Schmidt, we know that $W$ and $W^{\perp}$ each has an ____(a)____ basis. Let's say $(w_1, \ldots, w_m)$ and $(u_1, \ldots, u_k)$ are such bases for $W$ and $W^{\perp}$ respectively, where $m$ is the dimension of ___(b)___ and $k$ is the dimension of ___(c)___. Now, as $W + W^{\perp} = \mathbb{R}^n$, we have $m + k \geq n$. Also, we note that $(w_1, \ldots, w_m, u_1, \ldots, u_k)$ is an orthogonal list of vectors because ________(d)________. Hence, it is a linearly independent set of vectors in $\mathbb{R}^n$ and hecne $m + k \leq n$. Thus, we conclude $m + k = n$, as desired. □

*Problem 3.* (1 point) Answer the following questions: Did you find theory and examples lectured helpful? How helpful (say compared to the main lectures)? Did you look at the notes taken by the designated scribes? Is there anything you'd like to be changed for the section (e.g. more problem-solving, more theory, etc.)?

#2. (a) orthogonal

(b) $W$

(c) $W^{\perp}$

(d) w's and u's are orthogonal, and $W^{\perp}$ is the orthogonal vectors to $W$.

#3 Say something I'm giving up on you~