

Quiz #7; Wed, 3/9/2016

Math 53 with Prof. Stankova

Section 107; MWF10-11

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Problem. Consider the surface S that is a hyperboloid of 2 sheets given by $z^2 - 4 = x^2 + y^2$, and a point $P(2, 2, 2\sqrt{3})$ on the hyperboloid.

- (a) Find the tangent plane to S at the point P as follows: since z -coordinate of P is positive, write $z = \sqrt{x^2 + y^2 + 4}$ and now carry out the usual computation. (10 points)
- (b) Find the unique point Q (different from P) on S such that the tangent plane at Q is parallel to the tangent plane at P . (5 points)

Solution. (a) $\partial z/\partial x = (x)(x^2 + y^2 + 4)^{-1/2}$, $\partial z/\partial y = (y)(x^2 + y^2 + 4)^{-1/2}$. And so the equation for the plane is

$$z - 2\sqrt{3} = \frac{1}{\sqrt{3}}(x - 2) + \frac{1}{\sqrt{3}}(y - 2)$$

(b) By symmetry, one can guess $Q = (-2, -2, -2\sqrt{3})$. Indeed, writing $z = -(x^2 + y^2 + 4)^{1/2}$ (negative since z -coordinate is negative now), we get a parallel normal vector to part (a).