Quiz #3; Wed, 2/10/2016

Math 53 with Prof. Stankova

Section 110; MWF11-12

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Problem. Let L_1 be a line in $\mathbb{R}^3$ defined by the parametric equation: $x = t, y = 0, z = 2$. Find all pairs of numbers (z_0, c) such that the line L_2 defined by the symmetric equation $x = y = \dfrac{z - z_0}{c}$ *meets* L_1 **and** the two lines form an *acute angle of* $\pi/3$.

Solution. If the two lines intersect, we have $y = 0$, which by $x = y$ means $x = 0$, and hence $z - z_0 = 0$ as well, but $z = 2$ so $z_0 = 2$. Now, for the angle, the direction vector for L_1 is $\langle 1, 0, 0 \rangle$, and so $\dfrac{\langle 1,0,0 \rangle \cdot \langle 1,1,c \rangle}{\sqrt{1}\sqrt{1+1+c^2}} = \dfrac{1}{2}$ gives us $2 = c^2$ so $c = \pm\sqrt{2}$. Hence, the two pairs are $(z_0, c) = (2, \sqrt{2}), (2, -\sqrt{2})$.